Math 54

PROFESSOR KENNETH A. RIBET

Second Midterm Examination

November 2, 2005

2:10–3:00 PM

Name:

GSI's Name:

SID:

Special Status (e.g., Math 49 or Concurrent Enrollment):

Please put away all books, calculators, electronic games, cell phones, pagers, .mp3 players, PDAs, and other electronic devices. You may refer to a single 2-sided sheet of notes. Your paper is your ambassador when it is graded. Correct answers without appropriate supporting work will be regarded skeptically. Incorrect answers without appropriate supporting work will receive no partial credit. This exam has six pages. Please write your name on each page. At the conclusion of the exam, please hand in your paper to your GSI.

Problem:	Your score:	Total points
1		6 points
2		7 points
3		5 points
4		5 points
5		7 points
Total:		30 points

1. Let $R = \begin{bmatrix} 2 & 8 & -1 & 1 & 0 \\ 0 & 6 & -1 & 2 & -3 \\ 0 & 0 & -1 & -3 & -10 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$. Exhibit bases for the following three spaces:

- the row space of R,
- the column space of R,
- the null space of R.

2. Find three linearly independent eigenvectors for the matrix $\begin{bmatrix} 3 & 0 & -1 \\ 0 & 2 & 0 \\ -1 & 0 & 3 \end{bmatrix}$, whose characteristic polynomial is $(\lambda - 4)(\lambda - 2)^2$. Is this matrix diagonalizable?

3. Let W be the span of the three vectors $v_1 = (1, -1, 3, -2)$, $v_2 = (1, 9, 1, -10)$ and $v_3 = 2v_1 - v_2$ in $\mathbb{R}^4$. What is the dimension of W? Find an orthogonal basis for W.

4. Evaluate the determinant of the matrix $\begin{bmatrix} 1 & 0 & 0 & 2 & 4 & 6 & 8 \\ 0 & 1 & 0 & 5 & 12 & 13 & 9 \\ 0 & 0 & 1 & -1 & 31 & 5 & 23 \\ 0 & 0 & 0 & 4 & 2 & 7 & 1 \\ 0 & 0 & 0 & -2 & 1 & 3 & -2 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 2 & 5 & 3 \end{bmatrix}$.

5. Let A be an $n \times n$ (square) matrix. Suppose that $A^2 = A$. Show that $Ay = y$ for all y in the column space of A. If the null space of A is $\{0\}$, show that A is the identity matrix of size n.