Math 54, Spring 2008 **Midterm, March 7, 2008** **Chorin**

1. Define: Linear independence, rank of a matrix A, basis (of a subspace), span of $\underline{v}_1, \underline{v}_2, \underline{v}_3$.

2. Let A be the matrix
$$A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 0 & 1 & 1 \end{pmatrix}.$$
Find A^{-1} and $\det A$.

3. Let A be the matrix
$$\begin{pmatrix} 1 & 0 & 2 & 3 \\ 2 & -1 & 0 & 1 \end{pmatrix}.$$
Find a basis for the null space of A. Explain how you know it is a basis.

4. (i) Give an example of non-zero matrices A, B whose product is zero.
   (ii) Show that if A, B are square matrices, the dimension of the null space of B is zero, and $BA = 0$, then $A = 0$.

5. (i) Let T be a transformation that maps the vectors $(1,0,0)^T$ and $(0,1,0)^T$ on $(1,2,0)^T$, and maps $(0,0,1)^T$ on $(1,0,1)^T$. Find the matrix that performs T.
   (ii) Let $\mathcal{B}$ the unit ball $x_1^2 + x_2^2 + x_3^3 \leq 1$; find the volume of the image of $\mathcal{B}$ under T.