University of California at Berkeley  
College of Engineering  
Dept. of Electrical Engineering and Computer Sciences  

EE 105 Midterm II

Fall 2005                  Prof. Borivoje Nikolić                  November 17, 2005

Your Name: ____________________________

Student ID Number: _______________________

Guidelines

Closed book and notes; there are some useful formulas in the end of the exam.  
You may use a calculator.  
You can un staple the pages with formulas, but do not un staple the exam.  
Show all your work and reasoning on the exam in order to receive full or partial credit.  
Time: 80 minutes = 1 hour, 20 minutes.

Score

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1. MOS current source [20 points]
For the current source shown in Figure 1, dimensions of MOS transistors M1 and M2 are 
\((W/L)_1 = 2, (W/L)_2 = 10\). \(V_{DD} = 2.5\)V. \(\mu_n C_{ox} = 100\ \mu A/V^2\), \(V_{T0} = 0.5\)V, \(\lambda_n = 0.05\ V^{-1}\), 
\(\gamma = 0\).

a) [4 points] Find the value of \(R_{REF}\) such that \(I_{REF} = 100\mu A\). You can ignore channel 
length modulation in this part.

\[
\frac{V_{DD} - V_{GS1}}{R_{REF}} = 100\mu A
\]

\[
100\mu A = \left(\frac{W}{L}\right)_1 \cdot \mu_n C_{ox} \cdot \frac{(V_{GS1} - V_T)^2}{2}
\]

\[
(V_{GS1} - V_T)^2 = \frac{100\mu A \cdot 2}{2} \cdot \frac{1}{100\mu} = 1
\]

\[
V_{GS} = 1 + 0.5 = 1.5\ V
\]

\[
R_{REF} = \frac{2.5 - 1.5}{100\mu} = 10\ k\Omega
\]

\[
R_{REF} = 10\ k\Omega
\]
(b) [4 points] Find the value of $R_1$ that gives $I_{OUT} = 40\mu A$. Assume that $M_2$ is in saturation. You can ignore channel length modulation in this calculation.

$$V_{S2} = 40\mu A \quad \text{and} \quad 40\mu A = \left(\frac{V}{L}\right) \frac{\mu A \cdot \text{cm}}{\mu m} \left(V_{GS2} - V_t\right)^2$$

$$\Rightarrow \left(V_{G2} - V_{S2} - V_t\right)^2 = \frac{40}{10} \times \frac{2}{100}$$

$$\Rightarrow (1.5 - V_{S2} - 0.5)^2 = 0.08$$

$$\Rightarrow (1 - V_{S2})^2 = 0.08$$

$$\Rightarrow V_{S2} = 0.717V$$

So $R_1 = \frac{V_{S2}}{40\mu A} = 17.9k\Omega$

(c) [4 points] Find the lowest output voltage $V_{OUT}$ for which the circuit in Figure 1 still acts as a current source. You can ignore channel length modulation.

Condition for saturation

$V_{DS} > V_{GS} - V_{th}$

$\Rightarrow V_D > V_G - V_{th}$

$V_D = V_{OUT}$

$V_{OUT, min} = V_G - V_{th} = 1.5 - 0.5 = 1V$
(d) [6 points] Find the small-signal output resistance of the current source in Figure 1.

\[ V_{os} = -V_s \quad (V_o = 0 V) \]

\[ \text{It} = \frac{V_s}{R_1} \quad (1) \quad \text{It} = -V_s \cdot g_m + \frac{V_t - V_s}{n_0} \quad (2) \]

Replacing the value of \( V_s = \text{It} \cdot R_t \) in (2), we get

\[ \text{It} \left( 1 + g_m R_t + \frac{R_t}{n_0} \right) = \frac{V_t}{n_0} \]

\[ \text{R}_{out} = \frac{n_t}{n_0} = n_0 \left( 1 + g_m R_t + \frac{R_t}{n_0} \right) \]

\[ n_0 = \frac{1}{\lambda I_D} = \frac{1}{0.05 \times 100 \mu} = 500 \text{ kA}. \]

\[ g_m = \left( \frac{W}{L} \right) \mu \text{CoX} (V_{os} - V_t) = 10 \times 100 \mu \cdot (1.5 - 0.717 - 0.5) = 0.283 \text{ mS} \]

\[ R_t = 17.9 \text{ k} \Omega \]

\[ \text{So} \quad \text{R}_{out} = 500 \times 10^3 \left( 1 + 0.283 \times 10^{-3} \times 17.9 \times 10^3 + \frac{17.9 \times 10^3}{500 \times 10^3} \right) = 3.05 \text{ M} \Omega \]

\[ R_{out} = 3.05 \text{ M} \Omega \]
(e) [2 points] If body effect parameter, $\gamma > 0$, would it increase or decrease the value of output resistance from part (d)? Explain your answer.

Increase

$V_{tn} = V_{t0} + \gamma (V_{SB} - 2\Phi_p - \sqrt{-2\Phi_p})$

$V_{tn} \uparrow \Rightarrow I_0 \downarrow. \quad r_o = \frac{1}{\Delta I_0}$

$R_{out} \propto g_m \sqrt{I_d}$ so $g_m \downarrow$ but slower than $r_o$ increases.

so $R_{out} \propto g_m r_o \uparrow$.

$g_m \rightarrow g_m + g_m b$.

$R_{out} = r_o [(g_m + g_m b) R_1 + 1] + R_1$.

so $R_{out} \uparrow$.

$R_{out}$ increases / decreases (circle one)
2. MOS amplifiers [14 pts]
For the MOS amplifier in Figure 2, \((W/L)_1 = 10, (W/L)_2 = 20, (W/L)_3 = 10\), \(I_{\text{BIAS}} = 50\mu A\), 
\(V_{DD} = 2.5\) V. \(\mu_n C_{ox} = 100 \mu A/V^2, \mu_p C_{ox} = 30 \mu A/V^2, V_{th} = V_{TP} = 0.5\) V, \(\lambda_n = \lambda_n = 0.05 V^{-1}\),
\(\gamma = 0\). \(C_{GS1} = 2C_{GS1} = 2C_{GS3} = 100\) fF. \(C_{GD2} = 2C_{GD1} = 2C_{GD3} = 10\) fF.
Input voltage \(v_S\) has negligible input resistance and contains a DC and an AC component.

(a) [2 points] Find the bias current of transistor \(M_1\).

\[
I_{M1} = I_{\text{BIAS}} \frac{(W/L)_2}{(W/L)_1} = I_{\text{BIAS}} \times 2 = 100\mu A
\]

\[
I_{M1} = (100) \mu A
\]
(b) [4 points] Find the small-signal voltage gain $A_v = \frac{v_{out}}{v_s}$.

\[ A_v = \frac{v_{out}}{v_s} = -g_{m1} \left( n_{o1} \| n_{o2} \right) \]

\[ n_{o1} = \frac{1}{\lambda n I_m} = \frac{1}{0.05 \times 100 \mu} = 200 \text{ k}\Omega \]
\[ R_{out} = n_{o1} \| n_{o2} = 100 \text{ k}\Omega \]

\[ n_{o2} = \frac{1}{\lambda p I_m} = \frac{1}{0.05 \times 100 \mu} = 200 \text{ k}\Omega \]

\[ g_{m1} = \sqrt{2 \left( \frac{\lambda}{L} \right) I_m \mu_n C_{ox} \cdot \sqrt{2 \times 10 \times 100 \mu \times 100 \mu} = 0.447 \text{ mS} } \]

So \[ A_v = -44.7 \frac{V}{V} \]

\[ A_v = -44.7 \frac{V}{V} \]
(c) [4 points] Find the maximum and the minimum voltage at the output of this amplifier.

For M1:

\[ I_{M1} = \left( \frac{W}{L} \right)_1 \mu \text{m}_{\text{ox}} \left( V_{GS1} - V_{th} \right)^2 \]

\[ \Rightarrow 100 \mu A = \frac{10}{2} \times 100 \mu \left( V_{GS1} - 0.5 \right)^2 \]

\[ \Rightarrow V_{G1} = 0.947 \quad V_{GS1} = V_{G1} \]

\[ V_{out, min} = V_{G1} - V_{th} = 0.474 \text{V} \]

For M2:

\[ I_{M2} = \left( \frac{W}{L} \right)_2 \mu \text{p}_{\text{ox}} \frac{C_{GSS}}{2} \left( V_{GSS} - \left| V_{Tp} \right| \right)^2 \]

\[ \Rightarrow 100 \mu A = \frac{20}{2} \times 30 \mu \left( V_{GSS} - 0.5 \right)^2 \]

\[ \Rightarrow V_{GSS} = 1.07 \text{V} \quad \Rightarrow V_{G2} = 2.5 - 1.07 = 1.42 \text{V} \]

\[ \Rightarrow V_{out, max} = V_{G2} + \left| V_{Tp} \right| = 1.42 + 0.5 = 1.92 \text{V} \]

\[ V_{out, max} = 1.92 \quad V; V_{out, min} = 0.474 \text{V} \]

(d) [4 points] Find the frequency of the dominant pole of this amplifier.

\[ T = R_{out} \times C_{out} = R_{out} \times \left( C_{gd1} + C_{gd2} \left( 1 - \frac{1}{A_v} \right) \right) \]

\[ \frac{1}{T} = \frac{1}{100 \text{k} \times (20 \text{f})} = 5 \times 10^8 \text{rad/s} \]

\[ \omega = 5 \times 10^8 \text{ rad/s} \]
3. Amplifier frequency response [18 points]
An amplifier has all of its poles and zeros in the left-hand frequency plane (it is a stable, minimum-phase system) and an amplitude frequency response, as shown in Figure 3.

![Diagram of amplitude frequency response]

Figure 3

a) [6 points] Write the transfer function that produces this response.

\[ H(j\omega) = \frac{j \frac{\omega}{10}}{(1 + j\frac{\omega}{10^3})(1 + j\frac{\omega}{10^6})^2} \]

b) [6 points] Draw the phase response that corresponds to this amplitude response.

![Diagram of phase response]
(c) [6 points] Write the transfer function that produces the response from Figure 4.

\[ H(j\omega) = \frac{j \omega}{10} \times 10^{12} \leq \omega_0^2. \]

**Figure 4.**

\[ H(j\omega) = \frac{j \omega}{10} \times 10^{12} \leq \omega_0^2. \]

\[ 20 \log|\Omega| = 20 \implies Q = 10. \]

\[ H(j\omega) = \frac{j \omega 10^{10}}{(1 + j \frac{\omega}{10^3})(j \omega^2 + j \omega 10^5 + 10^{12})} \]

*Note: denominator.*

\[ 1 + \left( \frac{j \omega}{\omega_0} \right)^2 + \left( \frac{j \omega \omega_0}{Q} \right) \frac{1}{\omega_0^2} \implies multiply \ by \ \omega_0^2 \ on \ numerator.*