Show derivations and mark results with box around them. Erase or cross-out erroneous attempts. Simplify algebraic results as much as possible! Mark your name and SID at the top of the exam and all extra sheets.

You may need the following integrals:

\[
\int_{0}^{\infty} \frac{1}{1 + \frac{s}{\omega_0}} \left( \frac{s}{\omega_0} \right)^2 \, df = \frac{\omega_0}{4}
\]

\[
\int_{0}^{\infty} \frac{1}{1 + \frac{s}{\omega_0} + \frac{s^2}{\omega_0^2}} \left( \frac{s}{\omega_0} \right)^2 \, df = \frac{\omega_0 Q}{4}
\]

\[
\int_{0}^{\infty} \frac{s}{1 + \frac{s}{\omega_0} + \frac{s^2}{\omega_0^2}} \, df = \frac{\omega_0 Q}{4}
\]
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<td>Q1</td>
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In the fully-differential amplifier above:

a. (10pts) Calculate the input referred offset voltage for \( \Delta V_{\text{TH}} = 2 \text{mV} \), \( \Delta (W/L)/(W/L) = 1\% \), \( \Delta R/R = 1\% \). M1 and M2 have \( V' = 200 \text{mV} \).

b. (10pts) Assume only the following mismatches exist (different than the part a): \( \Delta R/R = 1\% \), \( \Delta g_{mb}/g_{mb} = 1\% \). Calculate the CMRR. \( g_{mb} = 0.1 g_m \)

\[
\Delta v_o = \Delta I \cdot R + \Delta R \cdot I
\]
\[
\Delta I = I \cdot \frac{\Delta w/L}{w/L} + g_m \cdot \Delta V_{\text{TH}}
\]
\[
\Delta v_{in} = \frac{\Delta v_o}{g_m R}, \quad \frac{v_o}{v_{in}} = g_m \cdot R
\]
\[
\Delta v_{in} = \frac{R}{g_m R} \left[ I \cdot \frac{\Delta w/L}{w/L} + g_m \cdot \Delta V_{\text{TH}} \right] + \frac{\Delta R \cdot I}{g_m R}
\]
\[
\Delta v_{in} = \frac{I}{g_m} \cdot \frac{\Delta w/L}{w/L} + \Delta V_{\text{TH}} + \frac{I \cdot \Delta R}{g_m R}
\]
\[
\Rightarrow \Delta v_{in} = \frac{100 \text{mV} \cdot 0.01 + 2 \text{mV} + 100 \text{mV} \cdot 0.01}{2} \approx 4 \text{mV}
\]
b) \[
V_{od/\text{cm}} = \Delta g_{mb} \cdot R \cdot V_{cm} + \Delta R \cdot g_{mb} \cdot V_{cm}
\]

We ignore:

\[
\frac{\Delta R \cdot g_{mb}}{g_{mb} \cdot R \cdot \text{small}}
\]

\[
A_d/\text{cm} = \Delta g_{mb} \cdot R + \Delta R \cdot g_{mb} = \frac{V_{od}}{V_{cm}}
\]

\[
A_d = \frac{V_{od}}{V_{pd}} = g_{m} \cdot R
\]

\[
CMRR = \frac{Ad}{A_d/\text{cm}} = \frac{g_{m} \cdot R}{\Delta g_{mb} \cdot R + \Delta R \cdot g_{mb}} = \frac{1}{\frac{\Delta g_{mb}}{g_{m}} + \frac{\Delta R \cdot g_{mb}}{R \cdot g_{m}}}
\]

\[
g_{mb} = 0.1 \text{ gm} \quad \Rightarrow \quad \Delta g_{mb} = 0.001 \text{ gm}
\]

\[
CMRR = \frac{1}{0.001 + 0.001} = 500 = 54 \text{ dB}
\]
Question 2 (20pts)

In the comparator circuit shown above, $V_1$ and $V_2$ are initialized to $V_{ocm}$. Due to the implementation errors switch S2 introduces and offset voltage of $V_{os}$. Given the offset voltage $V_{os}$, what is the minimum $V_{in}$ that this comparator can successfully detect?

Small signal model:

- $V_2$ & $V_1$ are initialized to 0 : small-signal
- For $V_{os} < 0$, $I_{di}$ cancels $I_{d3}$. $I_{d1}$ must be smaller than $I_{d3}$ for the correct transition.
  $I_{d1} < I_{d3}$
  $V_{os}, gm_1 < gm_3 \cdot A \cdot V_{in}$

$V_{os}, \frac{gm_1}{gm_3 A} \cdot \frac{1}{V_{in}} \Rightarrow$ Pre-amp decreases the influence of the $V_{os}$.  

Question 3 (20pts)

\[ \Phi_{1\text{late}} \rightarrow 3V \rightarrow 0V \]

1.5V \[ \begin{array}{c}
\text{M2} \\
20/1
\end{array} \]

\[ C = 10\text{pF} \]

\[ \frac{Q_{eb}}{2} \]

\[ \text{M1} \]

\[ 20/1 \]

\[ \Phi_1 \rightarrow 3V \rightarrow \]

In the sampling circuit above, find the total sampling error for a sampling time of \( t_s = 25\text{ns} \). Assume fast gating and 50\% charge split. Hint: \( e^4 \approx 0.02 \).

Parameter:

\( V_{THN} = 1V \), \( \mu_nC_{ox} = 200\mu A/V^2 \), \( C_{ox} = 5fF/\mu m^2 \), \( C_{oi} = 0.2fF/\mu m \), \( C = 1pF \).

Assume square-law and ignore the body-effect.

Errors:

1. Settling error
2. Charge injection
3. Overlap cap: \( C_{gdL} \)

1. \[ V_{in} \quad R_2 \quad C \quad \Downarrow \quad \Rightarrow \quad C = C \cdot (R_1 + R_2) \]

\[ R_1 = \frac{1}{\mu_nC_{ox}(W/L)_1 \cdot \left( \frac{3V - V_{TH}}{V_{TH}} \right)} \quad = \quad 125 \Omega \]

\[ R_2 = \frac{1}{\mu_nC_{ox}(W/L)_2 \cdot \left( \frac{3V - 1.5V - V_{TH}}{V_{TH}} \right)} \quad = \quad 500 \Omega \]
\[ Z = 10 \text{pf} \times 6.25 \Omega = 62.5 \text{nsec} \]

\[ \text{Settling error:} \quad 1.5V \cdot \frac{1}{C} = 1.5 \times 10^{-4} V = 1.5 \times 10^{-4} \text{V} \]

\[ = 30 \text{mV//} \]

2) Charge injection: Bottom plate sampling \& we'll consider M1 only.

\[ Q_{ch} = W_1 \cdot L_1 \cdot C_{ox} (V_{gs} - V_{th}) \]

\[ = 20 \mu \text{m} \times 1 \mu \text{m} \times \frac{5 \text{fF}}{\mu \text{m}^2} \times (3 - 1) = 20 \times 5 \times 2 \times 10^{-12} \text{F} \cdot \text{V} \]

\[ = 200 \text{pF} \cdot \text{V} \]

\[ \text{Charge injection error:} \quad \frac{Q_{ch}}{2} \cdot \frac{1}{C} = \frac{200 \text{pF} \cdot \text{V}}{10 \text{pF}} \times \frac{1}{C} \]

\[ = 10 \text{mV//} \]

3) Overlap cap:

\[ \Delta V = \frac{C_{ol}}{C} \cdot 3V \]

\[ C_{ol} = 4 \times 10^{-3} \text{F} \]

\[ \Delta V = \frac{4 \times 10^{-3} \text{F} \cdot 3V}{10 \text{pF}} = 1.2 \text{mV//} \]

Total sampling error:

\[ = -30 \text{mV} + 10 \text{mV} + 1.2 \text{mV//} \]

\[ = -18.8 \text{mV//} \]

Charge injection and overlap cap increase \( V_{ce} \),
while settling error decreases it.
Question 4 (30pts)

In the OTA above, all channel lengths are the same, all parasitic caps are 0 and \(2I_{\text{bias1}}=I_{\text{bias2}}=I\).

a. **(10pts)** Calculate the ratio of \(W_{1a}/W_{2a}\) that sets the non-dominant pole to of the OTA to 4X the unity gain frequency \(\omega_0\) of the open loop OTA. You can assume the 2\(^{nd}\) pole in the Miller compensation is at \([-\text{gm of the 2}^{\text{nd}} \text{ stage}]/C_c\).

b. **(5pts)** Calculate the value of \(R_{ca}\), in terms if the appropriate gm, that sets the RHP zero to infinity.

c. **(10pts)** Assuming the amplifier is in the feedback loop shown below, estimate the total output voltage noise in terms of \(V^*\). For this part you can ignore the effects of the non-dominant pole and the noise from the second stage.

\[
\begin{align*}
\text{Input} & \quad V_i^+ & \quad C_s & \quad V_i^- & \quad C_s \\
\text{Output} & \quad V_o & \quad C_s & \quad V_o & \quad C_s
\end{align*}
\]

d. **(5pts)** Using the feedback amplifier in part c, find the slewing time \(t_{\text{slew}}\) when the amplifier is driven by a differential input of \(+8V_{p}^*\).
a) Miller compensations

\[ \omega_c = \frac{g_{m1}}{C_c} \quad \rho_2 = \frac{g_{m2a} \text{ (given)}}{C_c} \quad \frac{g_{m2a}}{C_c} = \frac{g_{m1a}}{4} \]

\[ g_{m1a} = \frac{1}{4} \quad g_{m1a} = \sqrt{1 / \omega_c^2} \text{max} I_{D1a} \]

\[ g_{m2a} = \frac{1}{\sqrt{1 / \omega_c^2 + \text{max} I_{D2a}}} \]

\[ \frac{g_{m1a}}{g_{m2a}} = \frac{1}{\sqrt{\omega_c^2/\omega_c^2 + \text{max} I_{D2a}}} = \frac{1}{4} \quad I_{D1a} = I/2 \quad I_{D2a} = I \]

\[ \frac{\omega_{1a}}{\omega_{2a}} = \frac{1}{16} \quad \Rightarrow \quad \frac{\omega_{1a}}{\omega_{2a}} = \frac{1}{8} \]

b) \[ z = \frac{g_{m2a}}{C_c} \quad \text{with} \quad R_2 \quad z = \frac{1}{\left(\frac{1}{g_{m2}} - R_2\right)C_c} \]

\[ z \to \infty \quad R_2 = \frac{1}{g_{m2}} \]

differential

\[ A = \frac{V_{in}}{V_{eq}} = \left(\frac{1}{g_{m1}}\right)^2 \cdot \frac{4kT}{\Delta f} \cdot \frac{1}{\left(\frac{1}{g_{m1}} + g_{m1} + g_{m2}\right)x} \]

In the given feedback configuration \( F = \frac{1}{2} \):

\[ \left(\frac{V_{in}}{\Delta f} \rightarrow V_{eq} \right)^2 = \left(\frac{1}{F}\right)^2 \cdot \left| \frac{1}{1 + \frac{5}{\text{w}} \cdot \text{W} \cdot F} \right|^2 \]

\[ \frac{V_{eq}^2}{\Delta f} = \left(\frac{1}{g_{m1}}\right)^2 \cdot \frac{4kT}{\Delta f} \cdot \left(\frac{1}{g_{m1}} + g_{m2} + g_{m3}\right) \cdot 2 \cdot \left(\frac{1}{F}\right)^2 \cdot \left| \frac{1}{1 + \frac{5}{\text{w}} \cdot \text{W} \cdot F} \right|^2 \]
Noise integral \( \frac{W_u F}{4C} = \frac{1}{4} \frac{gm_1}{Cc} \cdot F \)

\[ \text{Vol}_e^2 = 2 \left( \frac{1}{gm_1} \right) 4kT \gamma \alpha \left( 1 + \frac{gm_4}{gm_1} \right) \left( \frac{1}{F} \right) \cdot \frac{1}{4} \frac{gm_1 \cdot F}{Cc} \]

\[ \text{Vol}_e^2 = 2 \cdot \frac{kT}{Cc} \cdot \frac{1}{F} \cdot \gamma \left( 1 + \frac{gm_4}{gm_1} \right) \]

\[ \text{Vol}_e^2 = 2 \cdot \frac{kT}{Cc} \cdot \frac{1}{F} \cdot \gamma \left( 1 + 2 \cdot \frac{V_{io}^*}{V_{4a}^*} + \frac{V_{id}^*}{V_{5a}^*} \right) \]

\[ \text{d)} \]

\( \Delta Vol = 8 V_{io}^* \)  
\( (\text{Signal gain} = 1) \)

\( \Delta Vol_{-\text{linear}} = \frac{V_{io}^*}{F} = 2V_{io}^* \)  \( (F = \frac{1}{2}) \)

\( \Delta Vol_{-\text{slew}} = 6V_{io}^* \)

\( I_{slew} \) is limited by the folded cascade and it is \( \frac{I}{2} \)

\[ \frac{I}{2} \cdot tslew \cdot \frac{1}{Cc} = 6 \cdot V_{io}^* \Rightarrow tslew = 6 \cdot Cc \cdot \frac{V_{io}^*}{I/2 + I_{10a}} \]

\[ tslew = 12 \cdot \frac{Cc}{gm_0} \]

\[ tslew = \frac{12}{W_u} \text{ open loop (not the loop gain) } \]
Question 5 (10pts)

In the switch cap gain circuit above, there are two non-overlapping phases available: $\Phi_1$, $\Phi_2$. In order to have an inverting gain stage with offset cancellation, assign appropriate phases ($\Phi_1$ or $\Phi_2$) to switches in the circuit.