Guidelines

- Closed book and notes.
- Two pages of information sheets allowed.
- Total time = 90 minutes
(1) For the circuit shown in Fig. 1, W/L = 2 for both M1 and M2, \( \mu_{n} C_{ox} = 100 \, \mu A/V^2, \lambda = 0.05 \, V^{-1}, V_{Th} = 1V, V_{DD} = 5V. \)

a) [5 pt] Find the DC drain current at M2 when \( V_{OUT} = 3V. \) Use \( \lambda = 0 \) for this part.

b) [5 pt] Find the DC gate bias (\( V_G \)) of M2 such that the DC output voltage \( V_{OUT} = 3V. \) Use \( \lambda = 0 \) for this part.

c) [5 pt] Draw the small-signal equivalent circuit. Find the values of all circuit elements in the small signal circuit (e.g., \( g_m, r_0, \ldots \)).

d) [5 pt] Find the voltage gain, \( A_v = v_{out}/v_s. \)

e) [5 pt] Find the output resistance of the circuit (both expression and numeric value).

f) [5 pt] Find the input resistance, and construct the two-port model of this voltage amplifier.

(2) Consider the following circuit with \((W/L)_1 = 2, (W/L)_2 = 1, \mu_p C_{ox} = 50 \, \mu A/V^2, \mu_n C_{ox} = 100 \, \mu A/V^2, \lambda_n = \lambda_p = 0.05 \, V^{-1}, V_{Th} = 1V \) and \( V_{Tp} = -1V, V_{DD} = 5V : \)

a) [10 pt] The gate is biased at 2.5V DC. Show that both transistors are in saturation regime.

Find the expression and numeric value of small-signal voltage gain, \( A_v = v_{out}/v_s. \)

b) [10 pt] Find the maximum and the minimum voltage at the output of this circuit when both transistors stay in saturation regime.
Consider the following circuit with 3 PMOS transistors: \((W/L)_1 = 10\), \((W/L)_2 = 20\), \((W/L)_3 = 100\), \(\mu p C_{ox} = 50 \ \mu A/V^2\), \(\lambda = 0.05 \ V^{-1}\), \(V_{TP} = -1 \ V\), \(I_{REF} = 10 \ \mu A\), \(V_+ = 3 \ V\), \(V_- = -3 \ V\), \(R_L = 100 \ \Omega\).

a) [5 pt] Can you identify any functional block in this circuit (i.e., any portion of the circuit that performs a known function)? Replace that functional block, and draw a simplified circuit of the amplifier.

b) [10 pt] Find the expression of the voltage gain, \(A_v = \frac{v_{out}}{v_s}\), and then find its numerical value.

c) [10 pt] Find both the expression and the numeric value of the output resistance of the amplifier.

![Fig. 3](image)

The frequency response of an amplifier is shown in the figure below.

![Fig. 4A and 4B](image)

a) [8 pt] Find the transfer function of the frequency response shown in Fig. 4A.

b) [7 pt] This transfer function can be realized by the circuit in Fig. 4B. Draw the small-signal circuit that includes \(C_{gd}\). For simplicity, we will neglect \(C_{gs}\). Analysis of this circuit can be simplified by Miller approximation. Draw the simplified small-signal equivalent circuit. Show the Miller capacitances explicitly in terms of other circuit parameters.

c) [10 pt] The following parameters of the circuit are given: \(I_{BIAS} = 10 \ \mu A\), \(\lambda = 0.1 \ V^{-1}\), \(r_{oc} = \infty\) (ideal current source).

If the frequency response of the amplifier matches the transfer function shown in Fig. 4A, find the numeric values of the transistor parameters: \(C_{gd}\), \(r_0\), \(g_m\) and the circuit parameter, \(R_s\).
Some equations

Threshold voltage (NMOS)

\[ V_{Tn} = V_{FB} - 2\phi_p + \frac{1}{C_{ox}} \sqrt{2q\varepsilon_s N_a (-2\phi_p)} \]

\[ \phi_p = \frac{kT}{q} \ln \frac{N_a}{n_i} \]

\[ V_{Tn} = V_{Tn0} + \gamma \left( \sqrt{V_{SB} - 2\phi_p} - \sqrt{-2\phi_p} \right) \]

NMOS equations:

\[ I_D = 0, \quad V_{GS} < V_{Tn} \]

\[ i_D = \frac{W}{L} \mu C_{ox} \left( V_{GS} - V_{Tn} - \frac{V_{DS}}{2} \right) V_{DS} \left( 1 + \lambda V_{DS} \right), \quad V_{GS} > V_{Tn}, \ V_{DS} < V_{GS} - V_{Tn} \]

\[ i_D = \frac{W}{L} \mu C_{ox} \left( V_{GS} - V_{Tn} \right)^2 \left( 1 + \lambda V_{DS} \right), \quad V_{GS} > V_{Tn}, \ V_{DS} > V_{GS} - V_{Tn} \]

MOS capacitances in saturation

\[ C_{gs} = \frac{2}{3} W L C_{ox} + C_{ov} \quad C_{ov} = L_D W C_{ox} \]

MOS signal parameters:

\[ g_m = \frac{\partial i_D}{\partial V_{GS}} \left| _{V_{GS}=V_{Tn}} \right. = \mu C_{ox} \frac{W}{L} \left( V_{GS} - V_{Tn} \right) \left( 1 + \lambda V_{DS} \right) \]

\[ \approx \frac{\mu C_{ox}}{L} \left( V_{GS} - V_{Tn} \right) \]

\[ = \sqrt{2i_D \left( \frac{W}{L} \right) \mu C_{ox}} \]

\[ r_o = \left( \frac{\partial i_D}{\partial V_{DS}} \right) \left| _{V_{GS},V_{DS}} \right.^{-1} \approx \frac{1}{\lambda I_{DS}} \]

\[ g_{mb} = \frac{\partial i_D}{\partial V_{BS}} Q \left| _{V_{GS}} \right. = \frac{\gamma g_m}{2\sqrt{-V_{BS} - 2\phi_p}} \]
(a) Current determined by $I_{D1}$
\[ V_{gS1} = V_{dS1} = V_{dd} - V_{out} = 5 - 3 = 2 \text{V} \quad \text{When } V_{out} = 3 \text{V} \]
\[ I_{D2} = I_{D1} = \frac{W}{L} \cdot \frac{\mu_{n}C_{ox}}{2} \left( \frac{V_{gS1} - V_{Th}}{2} \right)^{2} = 2 \cdot \frac{100}{2} \cdot 1^{2} = 100 \mu A \]
(b) \[ I_{D2} = I_{D1} = \frac{W}{L} \cdot \frac{\mu_{n}C_{ox}}{2} \left( \frac{V_{gS2} - V_{Th}}{2} \right)^{2} = 100 \]
\[ V_{gS2} - V_{Th} = 1 \Rightarrow V_{gS2} = 2 \text{V} \]

(c) \[ g_{m1} = \mu_{n}C_{ox} \left( \frac{W}{L} \right) (V_{gS1} - V_{Th}) = 100 \cdot 2 \cdot 1 = 200 \mu S \]
\[ g_{m2} = \mu_{n}C_{ox} \left( \frac{W}{L} \right) (V_{gS2} - V_{Th}) = 200 \mu S \]
\[ R_{o1} = \frac{1}{g_{m1}} = \frac{1}{0.05 \cdot 100 \cdot 10^6} = 200 \text{KS} \]
\[ R_{o2} = \frac{1}{g_{m2}} = 200 \text{KS} \]

\[ V_{gS1} = 0 - V_{out} = -V_{out} \]
\[ V_{gS2} = V_{S} \]

(d) KCL at $D_2$:
\[ g_{m2} V_{S} + \frac{V_{out}}{R_{o2}} - g_{m1} (-V_{out}) + \frac{V_{out}}{R_{o1}} = 0 \]
\[ R_{o1} = R_{o2} = R_{o} \]
\[ AV = \frac{V_{out}}{V_{S}} = -\frac{g_{m2}}{g_{m1} + \frac{2}{R_{o}}} = -\frac{2 \times 10^{-4}}{2 \times 10^{-4} + \frac{2}{2 \times 10^{5}}} = -\frac{2}{2.1} = -0.95 \]

(e) \[ \text{Set } V_{S} = 0 , \quad V_{out} \Rightarrow V_{out} = V_{C} \]
\[ \Rightarrow V_{gS2} = 0 \]
\[
\text{\( \bar{I}_C = \frac{U_E}{R_1} + \frac{U_E}{R_2} - (-g_m U_C) \)}
\]

\[
\Rightarrow R_{out} = \frac{U_E}{\bar{I}_C} = \left[ g_m + \frac{2}{R_0} \right]^{-1} = \left[ 2.1 \times 10^{-4} \right]^{-1} = 4.76 \text{ k}\Omega
\]

(5) \( R_{in} = \infty \)

\[
R_{in} = \infty \\
R_{out} = \left[ g_m + \frac{2}{R_0} \right]^{-1} = 4.76 \text{ k}\Omega \\
A_v = -\frac{g_m}{\left[ g_m + \frac{2}{R_0} \right]} = -0.95
\]

(2) (a) \( V_{G1} = V_{DD} - V_S = 2.5 \text{ V} \) \quad \( V_{G2} = V_S - 0 = 2.5 \text{ V} \)

\[
-I_{D1} = \left( \frac{W}{L} \right)_1 \cdot \frac{m_{n} \alpha \chi}{2} \cdot (V_{G1} - |V_T1|)^2 \cdot (1 + \lambda_p V_{S1})
\]

\[
-I_{D2} = \left( \frac{W}{L} \right)_2 \cdot \frac{m_{n} \alpha \chi}{2} \cdot (V_{G2} - V_{Tn})^2 \cdot (1 + \lambda_n V_{S2})
\]

\[
V_{S1} = V_{DD} - V_{out} = 5 - V_{out}
\]

\[
V_{S2} = V_{out} - 0 = V_{out}
\]

\[
\lambda_n = \lambda_p = \lambda
\]

\[
-I_{P1} = I_{D2} \quad \text{since} \quad \left( \frac{W}{L} \right)_1 \cdot \frac{m_{n} \alpha \chi}{2} = \left( \frac{W}{L} \right)_2 \cdot \left( \frac{m_{n} \alpha \chi}{2} \right)
\]

\[
1 + \lambda (5 - V_{out}) = 1 + \lambda \cdot V_{out}
\]

\[
\Rightarrow V_{out} = 2.5 \text{ V}
\]

For \( M_1 \), \( V_{G1} - |V_{T1}| = 2.5 - 1 = 1.5 \text{ V} < V_{S1} = 2.5 \Rightarrow M_1 \text{ in Saturating}
\]

For \( M_2 \), \( V_{G2} - |V_{Tn}| = 2.5 - 1 = 1.5 \text{ V} < V_{S2} = 2.5 \text{ V} \Rightarrow M_2 \text{ in Saturating} \)
\[ V_{S1} = 0 - V_S = -V_S \]
\[ V_{S2} = V_S - 0 = V_S \]
\[ g_{m1} = \left( \frac{W}{L} \right)_1 U_n \text{Cox} (V_{S1} - V_{TH}) \]
\[ = 2.50 \cdot (2.5 - 1) = 150 \text{ mS} \]
\[ g_{m2} = \left( \frac{W}{L} \right)_2 U_n \text{Cox} (V_{S2} - V_{TH}) \]
\[ = 1.10 \cdot (2.5 - 1) = 150 \text{ mS} \]
\[ I_{DP1} = I_{DP2} = \left( \frac{W}{L} \right)_2 \frac{U_n \text{Cox}}{2} (V_{S2} - V_{TH})^2 \]
\[ = 50 \cdot 10^{-5} \cdot 2 = 112.5 \text{ mA} \]
\[ R_0 = R_{o2} = \frac{1}{\lambda I_D} = 178 \text{ k\Omega} \]

KCL at D_1 = D_2

\[ g_{m2} \cdot V_S + \frac{V_{out}}{R_{o2}} - g_{m1} (-V_S) + \frac{V_{out}}{R_{o1}} = 0 \]

\[ A_v = \frac{V_{out}}{V_S} = - \frac{g_{m1} + g_{m2}}{1 / R_{o1} + 1 / R_{o2}} = - \frac{1}{2} (g_{m1} + g_{m2}) \cdot R_0 = 26.7 \]

(b) **Maximum** Voltage occurs when 
M1 is on the verge of saturation:

\[ V_{S1} = V_{DD} - V_{out} = V_{S1} - |V_{TP}| \]
\[ = V_{DD} - V_S - |V_{TP}| \]

\[ \Rightarrow V_S = V_{out} - |V_{TP}| \]

\[ I_1 = (\frac{W}{L}_1) \frac{U_n \text{Cox}}{2} [V_{DD} - V_{out}]^2 = I_2 = (\frac{W}{L}_2) \frac{U_n \text{Cox}}{2} [(V_{out} - |V_{TP}|) - V_{TH}]^2 \]

\[ \Rightarrow V_{DD} - V_{out} = V_{out} - |V_{TP}| - V_{TH} \Rightarrow V_{out} = \frac{1}{2} (V_{DD} + |V_{TP}| + V_{TH}) = 3.5 \]

**Minimum** Voltage: M2 at verge of sat. \( \Rightarrow V_S = V_{out} + V_{TH} \)

\[ I_1 = I_2 \Rightarrow V_{DD} - (V_{out} + V_{TH}) - |V_{TP}| = V_{out} \]

\[ \Rightarrow V_{out} = \frac{1}{2} (V_{DD} - |V_{TP}| - V_{TH}) = 1.5 \]
(3) (a) M₁ and M₂ form current mirror

\[ I_{D2} = I_{REF} \cdot \left( \frac{W}{L} \right)_2 / \left( \frac{W}{L} \right)_1 = 2 I_{REF} = 20 \, mA \]

The resistance looking into \( M_2 = R_{02} \)

\[ V_{Sg} = (V_{out} - V_S) \]

KCL at \( S \):

\[ g_{m3}(V_{out} - V_S) + \frac{V_{out}}{R_{03} || R_{02}} = 0 \]

\[ \Rightarrow A_V = \frac{V_{out}}{V_S} = \frac{g_{m3}}{g_{m3} + \frac{1}{R_{03} || R_{02}}} \]

Use DC analysis to find \( g_{m3}, R_{02}, R_{03} \)

\[ I_{D3} = 20 \, mA = (W/L)_3 \cdot \frac{U_p C_{ox}}{2} \cdot (V_{Sg} - 1V_{TP})^2 = 100 \cdot \frac{50}{2} (V_{Sg} - 1V_{TP})^2 \, mA \]

\[ V_{Sg} - 1V_{TP} = \sqrt{\frac{40}{5000}} = 0.089 \]

\[ g_{m3} = (W/L)_3 \cdot \frac{U_p C_{ox}}{2} \cdot (V_{Sg} - 1V_{TP}) = \frac{2 I_{D3}}{V_{Sg} - 1V_{TP}} = 4.49 \, MS \]

\[ R_{03} = \frac{1}{\lambda I_{D3}} = \frac{1}{0.05 \cdot 20 \times 10^{-6}} = 1 \, M\Omega \]

\[ R_{02} = \frac{1}{\lambda I_{D2}} = 1 \, M\Omega \]
(c) Set $V_S = 0, \ V_{Sg3} = V_{out} - 0 = V_{out} \rightarrow U_{T}$

\[ U_T = \frac{U_T}{R_{02}||R_{03}} + g_m U_T \]

\[ \Rightarrow R_{out} = \frac{U_T}{U_T} = \frac{1}{g_m + \frac{1}{R_{02}||R_{03}}} = 2.2 \, \text{k}\Omega \]

(4) (a) $\omega_1 = 10^5, \ \omega_2 = 10^7$

\[ H(j\omega) = \frac{100}{(1+j\frac{\omega}{10^5})(1+j\frac{\omega}{10^7})} \]

(b) $\downarrow$ Miller Approximation

\[ G_{M1} = (1 + g_m R_0) \cdot C_{gd} \]

\[ C_{M2} = (1 + \frac{1}{g_m R_0}) \cdot C_{gd} \]
\( r_0 = \frac{1}{2I_{\text{Bias}}} = \frac{1}{0.1 \times 10 \times 10^6} = 1 \text{ M\Omega} \)

\[ T_1 = R_s C_m = R_s \cdot (1 + 9m r_0) \cdot C_{gd} \]

\[ T_2 = R_0 \cdot C_{gd} \]

\[ C_{gd} = \frac{T_2}{r_0} = \frac{W_p}{r_0} = \frac{10^{-7}}{10^6} = 10^{-13} \]

\[ 9m r_0 = 40 \text{dB} = 10^2 = 100 \]

\[ 9m = \frac{100}{10^6} = 10^{-4} = 100 \text{ M\Omega} \]

\[ T_1 = 10^5 \approx R_s \cdot (10^1) \cdot 10^{-13} \approx R_s \cdot 10^{-11} \]

\[ \Rightarrow R_s = 10^6 = 1 \text{ M\Omega} \]