University of California at Berkeley  
College of Engineering  
Dept. of Electrical Engineering and Computer Sciences  

EE 105 Midterm I  

Spring 2005  
Prof. Roger T. Howe  
March 2, 2005  

Your Name:  SOLUTIONS

Student ID Number: ________________

Guidelines

Closed book and notes; one 8.5" x 11" page (both sides) of your own notes is allowed.  
You may use a calculator.  
Do not unstaple the exam.  
Show all your work and reasoning on the exam in order to receive full or partial credit.  
Time: 80 minutes = 1 hour, 20 minutes.

Score

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1. MOSFET circuit [17 points]

\[ V_{SUP} = 2.5 \text{ V} \]

\[ V_{IN} \]

\[ I_D \]

\[ V_{OUT} \]

\[ I_S = 125 \mu \text{A} \]

(a) [3 pts.] Assuming that the transistor is operating in saturation, find an equation for the drain current \( i_D \) in terms of the input voltage \( V_{IN} \), the output voltage \( V_{OUT} \), and the device parameters. It is not necessary to substitute numerical values.

\[
\frac{1}{2} \mu_n C_{ox} \left( \frac{W}{L} \right) \left( V_{GS} - V_{TH} \right)^2 \]

\[
\frac{1}{2} \mu_n C_{ox} \left( \frac{W}{L} \right) \left( V_{IN} - V_{OUT} - V_{TH} \right)^2
\]

(b) [4 pts.] For \( V_{IN} = 1.5 \text{ V} \), (i) find the numerical value of the output voltage in Volts and (ii) verify that the transistor is saturated for this case.

\[
i_D = I_S \quad \text{since} \quad I_S = 0 \quad \text{and} \quad I_{OUT} = 0.
\]

\[
\frac{1}{2} \mu_n C_{ox} \left( \frac{W}{L} \right) \left( V_{IN} - V_{OUT} - V_{TH} \right)^2 = I_S = 125 \mu \text{A}
\]

\[
\frac{1}{2} \left( \frac{100 \mu \text{A/V}^2}{500 \mu \text{A/V}^2} \right) \left( V_{IN} - V_{OUT} - 0.5 \right)^2 = 125 \mu \text{A}
\]

\[
\left( V_{IN} - V_{OUT} - 0.5 \right)^2 = \frac{125 \mu \text{A}}{500 \mu \text{A/V}^2} = \frac{1}{4} \text{ V}^2
\]

\[
V_{IN} - V_{OUT} - 0.5 \text{ V} = 0.5 \text{ V}
\]

\[
V_{OUT} = V_{IN} - 1 \text{ V} \Rightarrow V_{OUT} = 1.5 \text{ V} - 1 \text{ V} = 0.5 \text{ V} \quad (i)
\]

**Test:**

\[
V_{DS} = V_{DD} - V_{OUT} = 2.5 \text{ V} - 0.5 \text{ V} = 2 \text{ V}
\]

\[
V_{DS_{sat}} = V_{DS} - V_{TH} = V_{IN} - V_{OUT} - V_{TH} = 1.5 \text{ V} - 0.5 \text{ V} = 0.5 \text{ V}
\]

\[
\therefore \quad V_{DS} > V_{DS_{sat}} \quad \text{saturation} \quad \checkmark \quad (ii)
\]
(c) [3 pts.] For $v_{IN} = 0.5$ V, (i) find the numerical value of the output voltage and (ii) identify the transistor's operating region.

\[ v_{IN} = V_{Tn} \; ; \; v_{OUT} \geq 0 \; V \Rightarrow \text{transistor must have } v_{GS} \leq V_{Tn} \Rightarrow \]

\[ v_{OUT} = 0 \; V \; (i) \; \text{and it is cutoff} \; (ii) \]

(d) [4 pts.] Sketch the output voltage $v_{OUT}$ as a function of the input voltage $v_{IN}$ over the range $0 \; V \leq v_{IN} \leq 2.5$ V on the graph below. Note: the current source $I_S$ only works for $v_{OUT} > 0$ V and is a short-circuit for $v_{OUT} = 0$ V.

![Graph showing the relationship between $v_{OUT}$ and $v_{IN}$](image)

(e) [3 points] For a DC input voltage $V_{IN} = 1.5$ V, find the numerical value of the transconductance $g_m$. Note that you need not have solved either parts (a) or (d) to solve this part.

\[ \text{Transistor is saturated} \Rightarrow g_m = \frac{2 I_d}{V_{GS} - V_{Tn}} = \frac{2 \times 1.25 \mu A}{1.5 \; V - 0.6 \; V} = \frac{2 \times 1.25 \mu A}{0.9 \; V} \]

\[ g_m = 500 \, \mu S \]
2. Integrated charge-storage element [17 points]

Given: The n region connected to electrode B is doped with phosphorus with $N_d = 2 \times 10^{16}$ cm$^{-3}$ and with boron ($N_a = 1 \times 10^{16}$ cm$^{-3}$). The p region connected to electrode A is doped with only boron ($N_a = 1 \times 10^{16}$ cm$^{-3}$). The permittivity of silicon is $\varepsilon_s = 1.035 \times 10^{-12}$ F/cm and the permittivity of oxide is $\varepsilon_{ox} = 3.45 \times 10^{-13}$ F/cm. The thin oxide has a thickness $t_{ox} = 100$ nm. The built-in potential of aluminum is $\Phi_{Al} = -360$ mV.

(a) [4 pts] Sketch the charge density in thermal equilibrium along the x axis (see location in the in the cross section above. Given: the width of the depletion region on the p-side of the junction is $x_{pd} = 140$ nm = 0.14 \mu m.

\[
\rho_b \left[ C/cm^2 \right] \left( \times 10^{-3} \right)
\]

\[
-2.5 -2 -1.5 -1 -0.5 -0.25 0 0.25 0.5 1 1.5 2
\]

\[
-x_{pd}
\]

\[
x, [\text{nm}]
\]

\[
p\text{-side: } \rho = +q \cdot N_d = (1.6 \times 10^{-19}) \times (2 \times 10^{16} \text{ cm}^{-3}) = -1.6 \text{ mC/cm}^2
\]

\[
n\text{-side: } \rho = +q \cdot (N_d - N_a) = (1.6 \times 10^{-19}) \times (2 \times 10^{16} \text{ cm}^{-3} - 1 \times 10^{16} \text{ cm}^{-3}) = +1.6 \text{ mC/cm}^2
\]
(b) [3 pts.] Find the numerical value of the junction capacitance $C_{\text{junction}}(0)$ between the 20 x 20 $\mu$m$^2$ n-type region and the underlying p layer in thermal equilibrium ($V_{BA} = 0$ V) in fF. Given: 1 fF = $10^{15}$ F. Hint: the information given in part (a) should be very useful.

$$C_{\text{junction}}(0) = \frac{\varepsilon_0 A}{\varphi_n - \varphi_p} = \frac{(0.03 \times 10^{-12} \text{F/m}^2)(400 \times 10^{-6} \text{m}^2)}{2.8 \times 10^{-3} \text{m}}$$

$$= 1.47 \text{fF} \quad \text{[should have made the area 10 x greater]}$$

(c) [4 pts.] Plot the junction capacitance versus $V_{BA}$ on the graph below. If you couldn’t solve part (b), you can assume that the thermal equilibrium capacitance is 1000 fF in order to do this part.

$$C_{\text{junction}} = \frac{C_{\text{junction}}(0)}{\sqrt{1 - V_{BA}/\Phi_n}}$$

$$\Phi_B = \Phi_n - \Phi_p = 0.72$$
(d) [3 pts.] Sketch the capacitance of the $20 \times 20 \mu m^2$ thin-oxide area as a function of the voltage $V_{AB}$ on the graph below. Given: due to oxide charges, the threshold voltage is $V_{Th} = 4$ V, the minimum capacitance of the structure is one-half the maximum capacitance and the thermal-equilibrium capacitance is three-quarters of the maximum.

\[
V_{FB} = -(\phi_m - \phi_i) = -(-360 \, mV - (-360 \, mV)) = 0
\]

\[
C_{max} = \frac{C_{ox}}{\frac{L_{ox}}{W_{ox}}} = \frac{(3.45 \times 10^{-13} \, F/cm)}{400 \times 10^{-6} \, cm} = \frac{10^{-6} \, cm}{-1800 \, \mu F}
\]

(e) [3 pts.] Sketch the capacitance $C_{ba}$ as a function of the voltage $V_{AB}$ on the graph below. Ignore the contribution of the overlap of the metal onto the thick-oxide regions.

\[
C_{ba} (0) = C_{junction} (0) + C_{thin \, oxide} (0) = 147 + 1380 \, \mu F
\]

\[
C_{ba} (4V) = 60 \, \mu F + 630 \, \mu F = 750 \, \mu F
\]

\[
C_{ba} (6V) = 60 \, \mu F + 630 \, \mu F = 750 \, \mu F
\]

should have been larger
3. IC resistors [16 points]

Process Sequence:
1. **Starting material**: boron-doped silicon wafer with a concentration of $2 \times 10^{17}$ cm$^{-3}$
2. Deposit a 0.2 μm (= 200 nm) thick SiO$_2$ layer
3. Pattern the oxide using the **Oxide Mask** (dark field) by etching it down to the silicon.
4. Implant phosphorus with dose $Q_d = 2 \times 10^{12}$ cm$^{-2}$ and anneal to form a 50 nm-thick phosphorus-doped regions where the silicon is exposed.
5. Spin on photoresist and pattern with the **Implant Mask** (clear field).
6. Implant phosphorus with dose $Q_d = 2 \times 10^{12}$ cm$^{-2}$ and then etch off the photoresist.
7. Anneal to activate the second implant; the phosphorus regions remain 50 nm thick.
8. Deposit a 200 nm-thick SiO$_2$ layer and pattern using the **Contact Mask** (dark field).
9. Deposit 200 nm of aluminum and pattern using the **Metal Mask** (clear field).
Given: mobilities for this problem are \( \mu_n = 800 \text{ cm}^2/(\text{Vs}) \) and \( \mu_p = 200 \text{ cm}^2/(\text{Vs}) \). The saturation electric field for electrons is \( E_{sat} = \frac{2 x 10^4}{\text{V/cm}} \) and their saturation velocity is \( v_{sat} = 10^7 \text{ cm/s} \). Count the “dogbone” contact areas as 0.65 square each for both resistors.

(a) [4 pts.] Sketch the cross section \( A-A' \) on the graph below after step 9. Identify all layers clearly.

(b) [4 pts.] What is the sheet resistance \( R_\square \) of the 0.2 \( \mu \text{m} \) long, 0.1 \( \mu \text{m} \) wide resistor?

\[
R_\square = \frac{\rho}{t} = \left( \frac{q \eta \mu_n t}{L} \right)^{-1} = \left( \frac{1.6 \times 10^{-19}}{1.6 \times 10^{-12} \times 2 \times 10^{17} \times 800 - 5 \times 10^4} \right)^{-1}
\]

\[
\eta = N_d - N_a = \frac{Qd}{t} - N_a
\]

\[
= \frac{2 \times 10^{12}}{5 \times 10^{-6}} = 4 \times 10^8 \text{cm}^{-3}
\]

\[-2 \times 10^{17} \text{cm}^{-3}
\]

\[
R_\square = 7800 \Omega/\square
\]

(c) [4 pts.] What is the maximum current \( I_{max} \) in \( A/A \) through the 0.4 \( \mu \text{m} \) long, 0.05 \( \mu \text{m} \) wide resistor?

\[
I_{max} = q \eta v_{sat} \Rightarrow I_{max} = q W t \eta v_{sat}.
\]

\[
\eta = \left( \frac{2Qd}{t} \right)^{-1} - N_a = 8 \times 10^{17} \text{cm}^{-3} - 2 \times 10^{17} \text{cm}^{-3} = 6 \times 10^{17} \text{cm}^{-3}
\]

\[
\text{double due from step 6}
\]

\[
I_{max} = (1.6 \times 10^{-19}) (5 \times 10^{-6} \times 5 \times 10^{-4}) (6 \times 10^{17}) (10^7) \text{ A}.
\]

\[
I_{max} = 24 \mu \text{A}.
\]
(d) [4 pts.] Plot the current-voltage curve between terminals 1 and 2 over the range indicated on the graph below.

\[ v_{12} [V] \]

\[ i_{12} [\mu A] \]

\[ R_T = R_0 (2 + 1.3) = 2.57 \, k\Omega. \]

\[ I_{max_T} = \frac{V_{max_T}}{R_T} = \frac{(1.6 \times 10^{-3})(10^{-5})(5 \times 10^{-6})}{(10^3)} = 16 \mu A. \]

\[ V_{max_T} = R_T \cdot I_{max_T} = (25.7 \, k\Omega)(16 \mu A) = 0.41 [V]. \]

\[ R_D = \left( \frac{1.6 \times 10^{-12} \cdot 6 \times 10^{17} \times 0.5 \times 10^{-6}}{1} \right) = 2600 \, \Omega. \]

\[ R_B = R_D (8 + 1.3) = 24.2 \, k\Omega. \]

\[ V_{max_B} = R_B \cdot I_{max_B} = (24.2 \, k\Omega)(24 \mu A) = 0.58 [V]. \]