Spring 2021 CEE 191 Midterm Exam

Due March 17 at 10:00am on bCourses

March 17, 2021

Instructions

1. The exam duration is 50 minutes. 10 extra minutes are allotted for scanning and submission.
2. Exam questions will be made available at 9:00am on March 17th under the Midterm assignment on the bCourses class page. The exam should be submitted to the same assignment no later than 10:00am the same day. Late submissions will not be accepted or graded.
3. It is your responsibility to make sure you can download from and upload to bCourses. For this reason, a practice submission assignment has been available.
4. Questions can be asked at the regular Zoom lecture link during the exam. Only language clarification questions will be answered.
5. Solve the questions neatly and methodically using the provided template. The template can be printed and scanned or used as a soft copy on a tablet device. If you do not have access to a printer or a tablet, you can also use your own paper for the solution, provided that your scanned solution is clear and legible.
6. This is an open notes examination. However, you are ONLY allowed to use the materials posted on bCourses as hard copies or as pdf files. No other material is allowed.
7. You are NOT allowed to use the Internet or any software on your computer during the exam.
   Exceptions: downloading the questions, uploading the answers, accessing files on CE191 bCourses page.
8. You are prohibited to consult with any individual (real or virtual) while solving this exam.
9. Clearly state assumptions you have made for any missing information.
10. Important: In the next page, provide the requested information in the designated spaces and carefully read and sign the Honor Pledge. If you cannot print this template, please copy and sign the pledge by hand.

Problem 1. (40 points)

You are the manager of a concrete producing plant looking to expand by purchasing new concrete mixers. You have a budget of \$200,000 and a choice between two possible mixers. Mixer 1 costs \$50,000 and produces 13 ft^3 of concrete per day while Mixer 2 costs \$20,000 and produces 8 ft^3 of concrete per day. However, due to a green initiative at your company, you'd also like to limit your daily energy consumption to 10 kWh per day. Machine 1 consumes 1 kWh per day while Machine 2 consumes 2 kWh of energy per day. You want to determine how many mixers of each type you should buy such that you maximize your daily concrete production while not exceeding your financial and energy budgets. The information is summarized in Table 1.

	Cost	Production capacity, ft^3/day	Energy consumption, kWh/day
Mixer 1	\$50,000	13	1
Mixer 2	\$20,000	8	2

Table 1: Concrete mixers information

This is an integer program that can be formulated as:

$$\begin{aligned} \max_{\mathbf{x}} \quad & \mathbf{c}^T\mathbf{x} \\ \text{s.t.} \quad & A\mathbf{x} \leq \mathbf{b} \\ & x_1, x_2 \in \mathbb{Z} \end{aligned}$$

1. Determine the matrices $\mathbf{c}, A, \mathbf{b}$ according to the formulation above. (10 points)
2. Plot the constraints on the graph below and shade in the feasible region. Label the points where the constraints intercept the axes. (5 points)
3. Solve graphically the LP relaxation of the problem, report the optimal solution $\mathbf{x}^*$, and mark it on the graph. Assume the small squares on the graph are of size 1x1. (5 points)
4. Branch on x_1 and determine the optimal solution and cost of each subproblems from the graph . (5 point)
5. Complete the branch and bound by branching on x_2 in any remaining active subproblems and determine the final optimal solution and cost of the integer problem. (10 point)
6. Draw the tree of subproblems as seen in class and in the notes. Write the optimal solution x_1^*,x_2^* and cost of each subproblem in its node and label each branch with its corresponding new constraint. (5 points)

Problem 2. (20 points)

Note: For the following 2 parts, consider only the first-order standard gradient descent seen in Lab 2. In other words, take steps in the direction of the negative gradient.

1. Consider the following nonlinear multivariable function:

$$f(x_1, x_2) = 3x_1^2 + \log(x_2) \tag{1}$$

   (a) Calculate the gradient of f with respect to $\mathbf{x} = [x_1, x_2]^T$. (5 points)

   (b) Perform gradient descent on f with a starting point $\mathbf{x}^0 = [1, 1]^T$ and a step size $\alpha = 0.1$. Report $\mathbf{x}^1$ and $\mathbf{x}^2$ (up to three significant figures), the values of the vector $\mathbf{x}$ after the first and second iteration of gradient descent. (10 points)

2. Next, consider minimizing the quadratic function:

$$g(x) = x^2 \tag{2}$$

   Consider performing gradient descent on g with a starting point $x = 0.5$ and a step size $\alpha = 1$. Will gradient descent converge to a minimum? Explain your answer. (5 points)

Problem 3. (40 points)

Consider the following optimization problem, where $\mathbf{x} = [x_1, x_2]^T$:

$$\begin{aligned} \min_{\mathbf{x}} \quad & f(\mathbf{x}) = x_1^2 + x_2^2 \\ \text{s.t.} \quad & (x_1+1)^2 + (x_2-1)^2 \leq 2 \\ & (x_1+1)^2 + (x_2+1)^2 \leq 2 \end{aligned}$$

Hint: The equation of a circle with radius r centered at (a, b) is given by $(x_1 - a)^2 + (x_2 - b)^2 = r^2$.

1. Sketch the feasible set and indicate the optimal point $\mathbf{x}^*$ on the graph. Assume the small squares on the graph are of size 0.2x0.2 and the ticks on the axes are at integer values. (10 points)
2. Formulate the Lagrangian of the problem. (5 points)
3. Formulate the KKT conditions. (10 points)
4. Solve the KKT conditions for the minimization problem above. Hint: Use part 1 to find a candidate solution.(5 points)
5. Solve the KKT conditions for the maximization problem with the same objective and constraints. Hint: Use part 1 to find a candidate solution.(5 points)
6. Are the KKT conditions necessary? Are they sufficient? (5 points)