# UNIVERSITY OF CALIFORNIA AT BERKELEY 

Physics 7C - Stahler
Spring 2021

MIDTERM EXAM

Please do all your work in this exam, in the blank spaces provided.

You must attempt all four problems. If you become stuck on one, go on to another and return to the first one later. Be sure to show all your reasoning, since partial credit will be allotted. No credit will be given for unjustified answers. Remember to circle your final answer.

Please complete the following. On each subsequent page, please write your SID in the upper right corner, where indicated.

## Full name:

## SID:

## Discussion section and GSI:

## Signature:

## Problem 1 (25 points)

Joanne, a typical Physics 7C student, does so many extra homework problems that she has become nearsighted. That is, the lens in each eyeball, which has width $W$, focuses distant objects at a spot (red dot in the sketch) situated $\Delta x$ in front of the retina, at the the back of the eye. A pair of glasses placed in front of her eyes corrects the problem, so that she now sees clearly.

(a) Find $f_{0}$, the uncorrected focal length of each lens in Joanna's eyes.
(b) Are the lenses in her glasses converging or diverging? Why?
(c) The glasses sit a distance $L$ in front of Joanna's eyes. What is $f_{1}$, the focal length of of each lens in her glasses, when her vision is perfectly corrected? Be sure to indicate the algebraic sign of $f_{1}$.
(d) Joanna's friend Maria looks at her to check out the new glasses. Does Maria see a real or virtual image of Joanna's eyes?

## Problem 2 (25 points)

When we see a star, it is shifted in angle because of the atmosphere. Let's explore this shift quantitatively. Suppose a ray of starlight enters the atmosphere at angle $\theta_{*}$ with respect to the normal. Here, at the top of the atmosphere, the depth $d=0$, and the index of refraction is $n(0)=1$. At the bottom of the atmosphere, where $d=D$, the index is $n(D)>n(0)$. The angle with respect to the normal here is $\theta(D)$, and the shift is $\theta(0)-\theta(D)$. We suppose that $n$ increases exponentially with depth:

$$
\begin{equation*}
n(d)=n(0) \exp (d / \Delta d), \tag{1}
\end{equation*}
$$

where $\Delta d<D$.

(a) We first find the dependence of the inclination angle $\theta$ on $n$. The righthand sketch shows the ray crossing two thin layers of atmosphere, with slightly different $n$ - and $\theta$ values. By applying Snell's law and ignoring second-order quantities, you should find that a certain quantity is conserved in passing from one layer to the next. What is this quantity?
(b Using your result from (a), find $\theta(D)$ in terms of $\theta(0), D$, and $\Delta d$.
(c) For the case $\Delta d \ll D$, what is $\theta(D)$ ? Sketch the path of the ray in this case for $\theta(0)=\pi / 4$.

## Problem 3 ( 25 points)

Two plane electromagnetic waves, both with wavenumber $k$, travel in the $x$-direction toward a converging lens with focal length $f$. A detector sits at the point $f$. At a fixed point $x$ in front of the lens, the $E$-vectors of the two waves are

$$
\begin{aligned}
& \vec{E}_{1}=E_{0} \hat{y} \cos (k x-\omega t) \\
& \vec{E}_{2}=E_{0} \hat{y} \cos (k x-\omega t+\phi),
\end{aligned}
$$

where $\phi$ is a constant phase angle.

(a) Let $I_{1}$ be the intensity (energy per area per time) that would be absorbed by the detector if only Wave 1 were present. What is $I_{1}$ ?
(b) Let $I_{\text {tot }}(0)$ be the total intensity absorbed when both waves are present and the phase angle $\Phi=0$. What is $I_{\text {tot }}(0) / I_{1}$ ?
(c) Let $I_{\text {tot }}(\pi)$ be the total intensity absorbed when there are two waves and $\phi=\pi$. Find $I_{\text {tot }}(\pi) / I_{1}$.
(d) Finally, let $I_{\text {tot }}(\pi / 3)$ be the total intensity absorbed when there are two waves and $\phi=\pi / 3$. Find $I_{\text {tot }}(\pi / 3) / I_{1}$.

## Problem 4 ( 25 points)

A rocket launches from the Earth with speed $\beta c$. The launch occurs at time $t=0$ according to a clock on Earth, and at time $\tau=0$ according to a clock on board the rocket. When the rocket clock reads $\tau=\tau_{1}$, the rocket ejects its booster (shown here in black) back toward the Earth. The ejection speed is $\beta^{\prime} c$, relative to the rocket. The booster eventually crashes into the Earth.

(a) Find $L_{1}$, the rocket's distance, as measured from the Earth, when it ejects the booster. Express $L_{1}$ in terms of $\beta, c$, and the (known) time $\tau_{1}$.
(b) At what Earth time $t_{2}$ does the booster crash into the Earth? Assume that the booster has traveled at constant speed since it was ejected from the rocket. Express your answer in terms of $\beta, \beta^{\prime}, c$, and $\tau_{1}$.
(c) Finally, as measured by the clock on board the rocket, at what time $\tau_{2}$ does the crash occur?

