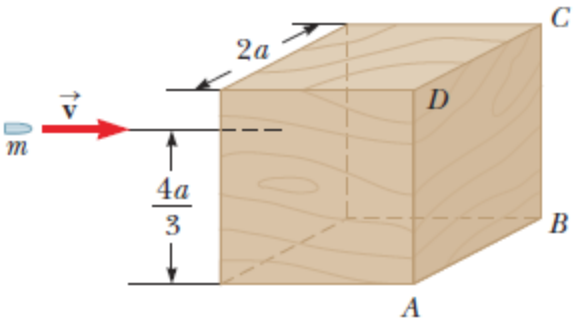
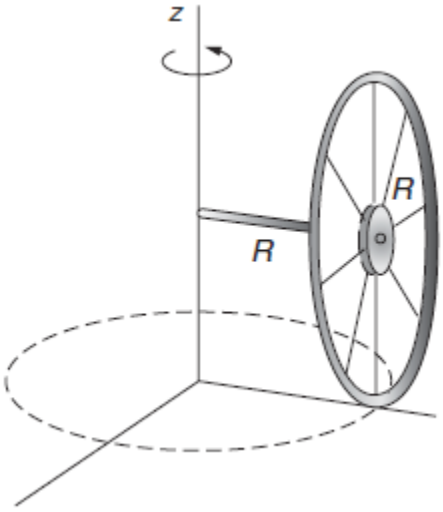
YILDIZ PHYSICS 7A Spring 2020 Final

1. (20 points) A solid cube of wood of side $2a$ and mass M is resting on a horizontal surface. The cube is constrained to rotate about a fixed axis AB. A bullet of mass m and speed v is shot at the face opposite $ABCD$ at a height of $4a/3$. The bullet becomes embedded in the cube. Find the minimum value of v required to tip the cube so that it falls on face $ABCD$. Assume $m << M$.

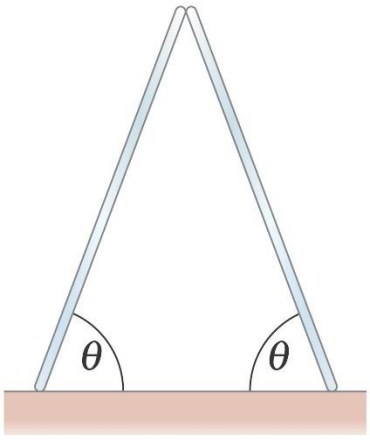


2. (20 points) A thin hoop of mass M and radius R rolls without slipping about the z-axis. It is supported by an axle of length R through its center, as shown. The hoop circles around the z-axis with angular speed Ω. In this case, the hoop *spins* around its own center of mass and it also *orbits* around the z-axis in a vertical position.

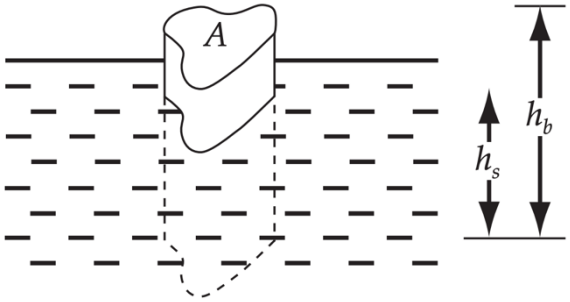


   a) What is the instantaneous spin angular velocity ω of the hoop around its center of mass?
   b) What is the moment of inertia of the orbital motion? (Hint: The hoop is vertically oriented in this case, it spins about its center of mass, and orbits around the z-axis).
   c) What is the spin angular momentum L of the hoop (i.e. angular momentum of the system about the axis passing through its center of mass perpendicular to z)?
   d) What is the orbital angular momentum L of the hoop (i.e. angular momentum of the system about the z-axis)?
   e) Bonus question: Is the total L vector parallel to the total ω vector?

3. (15 points) Two identical, uniform beams are symmetrically set up against each other on a floor with which they have a coefficient of friction $\mu_s = 0.5$. What is the minimum angle the beams can make with the floor and still not fall?

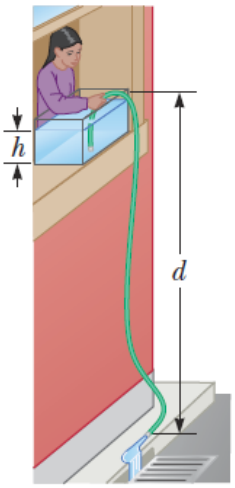


4. (15 points) A body of cross-sectional area of A, the height of h_b, and the density of ρ is partially submerged into a fluid with a density of ρ_0 ($\rho < \rho_0$).



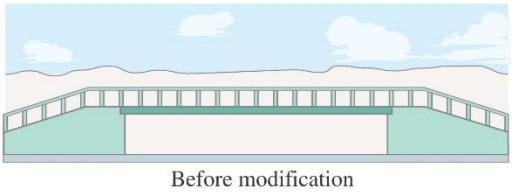
   a) Find the height h_s of the body submerged into the fluid at the equilibrium.
   b) If the body is displaced slightly away from the equilibrium, show that it will oscillate up and down about the equilibrium with simple harmonic motion. Assume that fluid is not viscous (i.e. no friction or drag forces).
   c) Find the period of this oscillation in terms of known quantities and g.

5. (15 points) A woman is draining her fish tank by siphoning the water into an outdoor drain. The rectangular tank has a footprint area A and depth h. The drain is located a distance d below the surface of the water in the tank, where $d >> h$. The cross-sectional area of the siphon tube is A'. Model the water as flowing without friction. What is the time interval required to empty the tank?

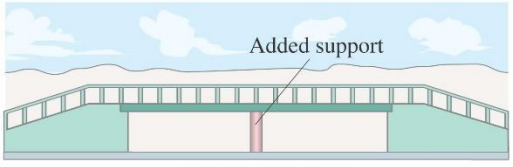


6. (15 points) A highway overpass of 150 m was observed to resonate at its fundamental harmonic when a small earthquake shook the ground vertically at 3 Hz.

a) What is the velocity of the earthquake wave?
b) The highway department put support at the center of the overpass, anchoring it to the ground. What resonant frequency would you now expect for the overpass?
c) It is noted that earthquakes rarely do significant shaking above 5 or 6 Hz. Did the modification do any good? Explain.



Before modification



After modification