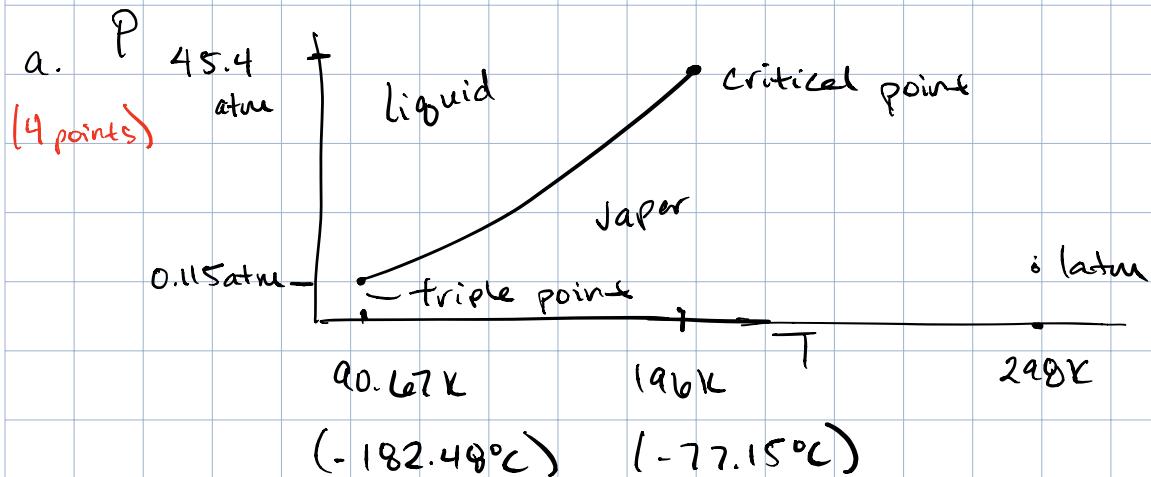


1. Natural gas storage



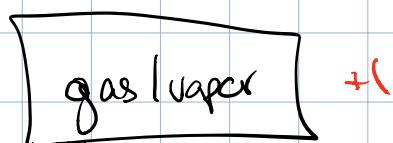
+1, correct triple point and critical point

+1, general PT diagram correct

+1, liquid vapor regions correct

methane's phase, ambient conditions ($T=298\text{K}$, $P=1\text{atm}$)

use graph



b) 623,000 moles $V=500\text{ m}^3$ $P=1\text{atm}$ (inert
(5 points)

$$P = \frac{nRT}{V} = \frac{(623,000)(8.314 \text{ J/mole})(298 \text{ K})}{500 \text{ m}^3}$$

+3, using ideal gas

$$P = 3087054 \text{ Pa.}$$

law

$$\boxed{P = 30.5 \text{ atm}}$$

Storage container can't
be used

+1 correct answer

+1 correct conclusion

$$c) T: -140^\circ\text{C} = 133.15 \text{ K}$$

$$i) P = \frac{nRT}{V} : \frac{(623,000)(8.314 \text{ J/moleK})}{500} / 133.15 \text{ K}$$

+1, using

ideal gas again

$$P = 1374333 \text{ Pa}$$

$$\text{if } T = -130^\circ\text{C}$$

$$P = 1482926 \text{ Pa}$$

$$P = 14.6 \text{ atm}$$

+1
+2

$$\boxed{P = 13.61 \text{ atm}}$$

if all vapor

$$P > P_{\text{max}}$$

check w/ Antoine's

+2, checking w/ Antoine's

$$P^* = 10^{\alpha} \left[b + \frac{c}{T - d} \right]$$

$$10^{\alpha} = 6.34159 - \frac{342.22}{-140 + 2100.221}$$

$$P^* = 3126 \text{ mmHg}$$

+1/2, correct P^*

+1, $P = P^*$, concluding tank is good

$$P^* = 4.11 \text{ atm}$$

$P^* < P_{\text{igL}}$, so vapor

single component: $P^* = P$

liquid equilibrium

this tank can be used!

if $T = -130^\circ\text{C}$ $P^* = 5171 \text{ mbarHg}$
 $P^* = 6.80 \text{ atm}$

same conclusions

ii.

$$\rho_{\text{liquid}} = 28100 \frac{\text{mol}}{\text{m}^3}$$

$$PV = nRT \quad \frac{P}{RT} = \frac{n}{V} = \rho = \frac{(4.11 \text{ atm})(101325 \frac{\text{Pa}}{\text{atm}})}{(8314 \frac{\text{J}}{\text{mol K}})(133.15)}$$

$$\rho = 376.19 \frac{\text{mol}}{\text{m}^3} \quad \begin{aligned} & \text{density of vapor phase} \\ & + 2, \text{solving density of } V \end{aligned}$$

+1 total volume = liquid volume + vapor volume

+1 mole balance

$$623,000 \text{ mol} = \left(376.19 \frac{\text{mol}}{\text{m}^3}\right) V_{\text{vapor}} + 28100 \frac{\text{mol}}{\text{m}^3} (V_{\text{liquid}})$$

$$500 \text{ m}^3 = V_{\text{vapor}} + V_{\text{liquid}} \quad V_{\text{vapor}} = 500 - V_{\text{liquid}}$$

+1 vapor balance

+1 solving system of equations

$$623000 \text{ moles} = \left(376.19 \frac{\text{mol}}{\text{m}^3} \right) (500 - V_{\text{liquid}})$$

$$+ 28100 V_{\text{liquid}}$$

$$623000 = 188045 - 376.19 V_{\text{liquid}} + 28100 V_{\text{liquid}}$$

$$434905 = 27723.81 V_{\text{liquid}}$$

$$V_{\text{liquid}} = 15.687 \text{ m}^3$$

$$V_{\text{vapor}} = 484.313 \text{ m}^3$$

$$\text{moles vapor} = 484.313 \text{ m}^3 \times 376.19 \frac{\text{mol}}{\text{m}^3} = 177835 \text{ moles}$$

$$\text{moles liquid} = 15.687 \text{ m}^3 \times 28100 \frac{\text{mol}}{\text{m}^3} = 440805 \text{ moles}$$

$$\boxed{\begin{aligned} \text{Mol vapor} &= 178 \text{ Kmol} \\ \text{Mol liquid} &= 441 \text{ Kmol} \end{aligned}}$$

* allows some rounding error

+2 final answers

If $T = -130^\circ\text{C}$ $P^* = 6.80 \text{ atm}$

$$\frac{P}{RT} = \frac{n}{V} = \frac{P_{\text{vapor}}}{P^*} = \frac{(6.80 \text{ atm})(101325 \text{ Pa/atm})}{(8.314 \text{ J/molK})(143.15)}$$

$$P_{\text{vapor}} = 578.93 \frac{\text{mol}}{\text{m}^3}$$

$$623000 = \left(578.93 \frac{\text{mol}}{\text{m}^3} \right) V_{\text{vapor}} + 28100 V_{\text{liquid}}$$

$$500 \text{ m}^3 = V_{\text{vapor}} + V_{\text{liquid}}$$

$$623000 = (578.93)(500 - V_{\text{liquid}}) + 28100 V_{\text{liquid}}$$

$$V_{\text{liquid}} = 12.12 \text{ m}^3 \quad V_{\text{vapor}} = 487.88 \text{ m}^3$$

| |
|-------------------------|
| Mol vapor = 282 Kmoles |
| Mol liquid = 341 Kmoles |

* allow some
rounding error

-2 if correct idea but math errors

$$+1 \qquad \qquad +1$$

$$\text{iii) } T_r = \frac{133.15}{196} = 0.679 \qquad P_r = \frac{4.11}{45.4} z = 0.091$$

($\frac{1}{2}$ credit if T_r found using $^{\circ}\text{C}$)

Use T_r and P_r to find z on graph

$$z \rightarrow 0.70 \quad (\text{approximately!}, \text{ almost not marked})$$

Methane non ideal here, on the saturation line,
 z indicates some compressibility.

An equation of state would be better to use.

+1, finding a close value of z and concluding
 somewhat non ideal

2.

$$F \text{ stream} = 1000 \text{ kmole/hr}$$

$$Z_H = 0.2$$

$$Z_p = 0.35$$

$$Z_o = 0.45$$

a)

For bubble point

$$\sum y_i = 1. + |$$

hep.

$$x_i p_i^{\text{sat}} = y_i p_{\text{sys}}$$

$$P_{\text{sat}} = 10^1 (6.90253 - \frac{1267.828}{T + 216.823}) + |$$

Pentane:

$$P_{\text{sat}} = 10^1 (6.84471 - \frac{1060.793}{T + 231.541}) + |$$

Oct:

$$P_{\text{sat}} = 10^1 (6.91874 - \frac{1351.756}{T + 209.106}) + |$$

$$\frac{0.2 P_H^{\text{sat}} + 0.35 P_p^{\text{sat}} + 0.45 P_o^{\text{sat}}}{1520 \text{ mmHg}} = 1 + 2 \quad \text{Solve for } T.$$

$$\text{bubble point } T = 91.6^\circ \text{C.} + |$$

dew point:

$$\sum x_i = 1 \quad +/ \quad \sum \frac{y_i p_{sys}}{p_i^{sat}} = 1$$

$$1520 \left(\frac{0.2}{p_H^{sat}} + \frac{0.35}{p_P^{sat}} + \frac{0.45}{p_o^{sat}} \right) = 1 \quad +/$$

solve for T.

$$\text{dew point temp} = 132.0 {}^{\circ}\text{C}$$

+ /

b). Version 1.

$$T = 120 {}^{\circ}\text{C} \quad P = 2 \text{ atm} \quad F = 1000 \text{ kmol/h}$$

Find $y_H, y_P, y_o, D; x_H, x_P, x_o$ and B.

Rachford-Rice eq.

$$x_i = \frac{z_i}{1 + \left(\frac{V}{F} \right) \left(\frac{p_i^{sat}}{p_{sys}} - 1 \right)}$$

$$\sum x_i = \sum \frac{z_i}{1 + \frac{V}{F} \left(\frac{p_i^{sat}}{p_{sys}} - 1 \right)} = 1$$

p_H^{sat} .

$$p_H^{sat} = 10^{-1} (6.90253 - \frac{1267.828}{120 + 216.823}) = 1375.473 \text{ mmHg} \quad +/$$

$$p_P^{sat} = 10^{-1} (6.84471 - \frac{1060.793}{120 + 231.541}) = 6716.743 \text{ mmHg} \quad +/$$

$$p_o^{sat} = 10^{-1} (6.91874 - \frac{1351.756}{120 + 209.1}) = 647.601 \text{ mmHg} \quad +/$$

$$1 = \left(\frac{0.2}{1 + \frac{V}{1000} \left(\frac{1375.437}{1520} - 1 \right)} + \frac{0.35}{1 + \frac{V}{1000} \left(\frac{6716.743}{1520} - 1 \right)} + \frac{0.45}{1 + \frac{V}{1000} \left(\frac{647.601}{1520} - 1 \right)} \right) + 2$$

Solve for V.

$$D = V = 572.60 \text{ kmol/hr.} + 2$$

$$B = 1000 - 572.60 = 427.40 \text{ kmol/h.} + 2$$

Now solve for x_i using Rachford-Rice eq.

$$x_h = \frac{0.2}{1 + \frac{572.60}{1000} \left(\frac{1375.437}{1520} - 1 \right)} = 0.211. + 1$$

$$x_p = \frac{0.35}{1 + \frac{572.60}{1000} \left(\frac{6716.743}{1520} - 1 \right)} = 0.118 + 1$$

$$x_o = \frac{0.45}{1 + \frac{572.60}{1000} \left(\frac{647.601}{1520} - 1 \right)} = 0.671 + 1$$

$$y_h = \frac{(x_h)(P_h^{\text{sat}})}{1520} = 0.191 + 1$$

$$y_p = \frac{(x_p)(P_p^{\text{sat}})}{1520} = 0.523 + 1$$

$$y_o = \frac{(x_o)(P_o^{\text{sat}})}{1520} = 0.286 + 1$$

For incorrect answers due to errors from a prior calculation but with correct work, only deduct 0.5 points per answer

B) Version 2.

$$B = 500 \text{ kmol/h} \quad P = 2 \text{ atm.}$$

Find T, y_H, y_P, y_O .

$$P_H^{\text{sat}} = 10^2 \left(6.90253 - \frac{1267.828}{T + 216.823} \right) + 1$$

$$P_P^{\text{sat}} = 10^2 \left(6.84471 - \frac{1060.793}{T + 231.541} \right) + 1$$

$$P_O^{\text{sat}} = 10^2 \left(6.91874 - \frac{1351.756}{T + 209.1} \right) + 1$$

Rachford - Rice eq

$$\sum x_i = \sum \frac{z_i}{1 + \frac{v}{f} \left(\frac{p_i^{\text{sat}}}{p_{sys}} - 1 \right)} = 1$$

$$1 = \frac{0.2}{1 + \frac{1}{2} \left(\frac{p_H^{\text{sat}}}{1520} - 1 \right)} + \frac{0.35}{1 + \frac{1}{2} \left(\frac{p_P^{\text{sat}}}{1520} - 1 \right)} + \frac{0.45}{1 + \frac{1}{2} \left(\frac{p_O^{\text{sat}}}{1520} - 1 \right)} + 3$$

Solve for T.

$$T = 117.05 + 2$$

$$y_H = \frac{x_H p_i^{\text{sat}}}{p_{sys}} + 1$$

$$x_H = \frac{0.2}{1 + \frac{1}{2} \left(\frac{1274.04}{1520} - 1 \right)} = 0.2176 + 1$$

$$y_H = 0.182 + 1$$

$$x_P = \frac{0.35}{1 + \frac{1}{2} \left(\frac{6333.257}{1520} - 1 \right)} = 0.1355 + 1$$

$$y_P = 0.565 + 1$$

$$x_O = \frac{0.45}{1 + \frac{1}{2} \left(\frac{594.5156}{1520} - 1 \right)} = 0.6469 + 1$$

$$y_O = 0.253 + 1$$

For incorrect answers due to errors from a prior calculation but with correct work, only deduct 0.5 points per answer

Part 2, version 1 and 2: only difference is pt. D

a. highest conversion? Dictated by equilibrium, when $r_A = 0$ +2

$$0 = -kP_A + kP_B \rightarrow P_A = P_B \text{ at equilibrium} \stackrel{\text{①}}{\rightarrow} P_A + P_B = P \text{ and } y_A P = P_0 \\ 2P_A = P$$

$$\text{so } y_A(2P_A) = P_A \rightarrow y_A = 0.5 \text{ at equilibrium}$$

Species A balance around reactor: $D = I - C$

$$n_{A,0} = y_{A,0} n_0 = y_{A,0} \dot{n}_1 = \dot{n}_1 - \dot{n}_1 f_A \rightarrow y_{A,0} = 1 - f_A \stackrel{\text{②}}{\rightarrow} \text{so } f_A = 0.5$$

\downarrow
Total mol
balance ($I=0$)
 $\dot{n}_0 = \dot{n}_1$

b. $r_A = -kP_A + kP_B$

$$\rightarrow y_A P = P_A, y_B P = P_B$$

$$\text{so, } r_A = kP(y_B - y_A) \stackrel{\text{①}}{\rightarrow}$$

$$\rightarrow y_B + y_A = 1 \rightarrow y_B = 1 - y_A \stackrel{\text{②}}{\rightarrow}$$

$$\text{so, } r_A = kP(1-y_A - y_A) = \boxed{kP(1-2y_A) = r_A}$$

c. $\frac{df_A}{dV} = -\frac{r_A}{\dot{n}_1} = -\frac{kP(1-2y_A)}{\dot{n}_1} \stackrel{\text{③}}{\rightarrow}$ relate y_A to f_A : $y_A = \frac{\dot{n}_{A,0}}{\dot{n}_1} \stackrel{\text{④}}{\rightarrow}$

$$\dot{n}_{A,0} = \dot{n}_1 - \dot{n}_1 f_A = \dot{n}_1 (1-f_A)$$

of moles does not change in reactor

$$\text{so } y_A = (1-f_A) \rightarrow \text{plug into PFR design eqn:}$$

$$\frac{df_A}{dV} = -\frac{kP}{\dot{n}_1} (1-2(1-f_A)) = -\frac{kP}{\dot{n}_1} (2f_A - 1) = \frac{kP}{\dot{n}_1} (1-2f_A)$$

Rearrange:

$$\int \frac{df_A}{1-2f_A} = \frac{kP}{\dot{n}_1} \int dV \rightarrow -\frac{1}{2} \ln(1-2f_A) = \frac{kP}{\dot{n}_1} \cdot V + C \stackrel{\text{⑤}}{\rightarrow}$$

1c. (cont): ② $V=0L, f_A=0 \rightarrow -\frac{1}{2} \ln(1-2(0)) = \frac{kP(0)}{n_1} + C \rightarrow C=0$ (+1)

$$\text{so } \ln(1-2f_A) = -2 \frac{kP}{n_1} V$$

$$f_A = \frac{1}{2} - \frac{1}{2} \exp(-2 \frac{kP}{n_1} V) \quad (+1)$$

$$V = -\frac{1}{2} \frac{n_1}{kP} \cdot \ln(1-2f_A)$$

d. (version 1) V needed to achieve $f_A = 0.25$, w/ $n_1 = 100 \frac{\text{mol}}{\text{s}}$, $k = 0.01 \frac{\text{mol}}{\text{L s atm}}$, $P = 10 \text{ atm}$

$$V = -\frac{1}{2} \frac{100 \frac{\text{mol}}{\text{s}}}{0.01 \frac{\text{mol}}{\text{L s atm}} \cdot 10 \text{ atm}} \ln(1-2 \cdot 0.25) = 347 \text{ L} = V \quad (+2) \quad (+2) \quad (-2 \text{ for incorrect answer})$$

d (version 2) f_A if $V = 500 \text{ L}$, $n_1 = 100 \frac{\text{mol}}{\text{s}}$, $k = 0.01 \frac{\text{mol}}{\text{L s atm}}$, $P = 10 \text{ atm}$

$$f_A = \frac{1}{2} - \frac{1}{2} \exp\left(-2 \frac{(0.01)(10)}{100}(500)\right) = 0.316 = f_A \quad (+2) \quad (+2) \quad (-2 \text{ for incorrect answer})$$

e. \dot{n}_2 and \dot{n}_3 in terms of \dot{n}_1 ?

Balance around reactor: $I=0 \rightarrow \dot{n}_2 = \dot{n}_3$ (+1)

split balance: $I=0: \dot{n}_3 = \dot{n}_4 + \dot{n}_5 = 2\dot{n}_4$ (+1)

mix balance: $I=0: \dot{n}_1 + \dot{n}_4 = \dot{n}_2$ (+1)

Combine reactor and split: $\dot{n}_2 = \dot{n}_3 = 2\dot{n}_4$

$$\dot{n}_1 + \dot{n}_4 = \dot{n}_2 = \dot{n}_1 + \frac{\dot{n}_2}{2} = \dot{n}_2 \rightarrow \dot{n}_2 = 2\dot{n}_1 = \dot{n}_3 \quad (+2)$$

f. $\dot{n}_{2,A} = f(n_1 \text{ and } f_A)$?

$$\text{Reactor balance: } \dot{n}_{3,A} = \dot{n}_{2,A} - \dot{n}_{2,A}f_A = \dot{n}_{2,A}(1-f_A) \quad (0=I-C) \quad (+1)$$

$$\text{Split balance: } \dot{n}_{3,A} = \dot{n}_{4,A} + \dot{n}_{5,A} = 2\dot{n}_{4,A} \quad (I=0) \quad (+1)$$

$$\text{Mix balance: } \dot{n}_1 + \dot{n}_{4,A} = \dot{n}_{2,A} \quad (I=0) \quad (+1)$$

$$\text{combine Reactor and split: } 2\dot{n}_{4,A} = \dot{n}_{2,A}(1-f_A) \quad [1]$$

$$\text{Combine [1] and mix: } \dot{n}_1 + \frac{1}{2}\dot{n}_{2,A}(1-f_A) = \dot{n}_{2,A} \quad (+2)$$

Rearrange to solve for $\dot{n}_{2,A}$:

$$\boxed{\dot{n}_{2,A} = \frac{2\dot{n}_1}{(1+f_A)}}$$

g. $y_{A,3} = \frac{\dot{n}_{3,A}}{\dot{n}_3} \quad (+2)$

from e), $\dot{n}_3 = 2\dot{n}_1$

$$\dot{n}_{3A} = n_{2A}(1-f_A) \text{ and } \dot{n}_{2A} = \frac{2\dot{n}_1}{1+f_A} \quad \text{from f)} \quad (+3)$$

$$\text{so: } y_{A,3} = \frac{\dot{n}_{3A}}{\dot{n}_3} = \frac{\dot{n}_{2A}(1-f_A)}{2\dot{n}_1} = \frac{2\dot{n}_1(1-f_A)}{2\dot{n}_1(1+f_A)} = \boxed{\frac{(1-f_A)}{(1+f_A)} = y_{A,3}}$$

h. $\frac{df_A}{dV} = -\frac{r_A}{\dot{n}_{2,A}} \quad (+1)$ $r_A = kP(1-2y_A)$, so: $\frac{df_A}{dV} = \frac{-kP(1-2\frac{(1-f_A)}{(1+f_A)}) \cdot (1+f_A)}{2\dot{n}_1} \quad (+3)$

$$\begin{aligned} &\hookrightarrow \dot{n}_{2,A} = \frac{2\dot{n}_1}{(1+f_A)} \\ &\text{for incorrect form} \quad (+2) \end{aligned} \quad \frac{df_A}{dV} = \frac{-kP}{2\dot{n}_1} \left(1+f_A - 2(1-f_A) \right) = \frac{-\frac{kP}{2\dot{n}_1} (3f_A - 1)}{dV} \quad (+5) \text{ for correct form}$$

$$\rightarrow -\frac{1}{3} \ln(1-3f_A) = \frac{kP}{2\dot{n}_1} V + C \quad \text{or} \quad \int \frac{df_A}{1-3f_A} = \frac{kP}{2\dot{n}_1} \int dV$$

$f_A = 0$ when $V = 0$ L, so $C = 0$

$$\therefore V = -\frac{2}{3} \frac{\dot{n}_1}{kP} \ln(1-3f_A) \quad (+2)$$

Part 2, Version 3 and 4: only difference is pt. D, 3/4 have a different rate expression
 than 1/2 ($r_A = -kP_A + \frac{1}{2}kP_B$ vs $r_A = -kP_A + kP_B$)

1-3/4

a) highest conversion dictated by equilibrium: $r_A = 0$ ②

$$0 = -kP_A + \frac{k}{2}P_B \rightarrow 2P_A = P_B \text{ at equilibrium} \rightarrow P_A + P_B = P, \text{ so } P_A + 2P_A = P \\ 3P_A = P$$

$$y_A P = P_A \rightarrow y_A (3P_A) = P_A, y_A = \frac{1}{3} \text{ at equilibrium}$$

species A balance around reactor: $0 = I - C$

$$\dot{n}_{A,0} = y_{A,0} \dot{n}_0 = y_{A,0} \dot{n}_1 = \dot{n}_1 - \dot{n}_1 f_A \rightarrow y_{A,0} = 1 - f_A$$

②
 Total mol
 balance ($I=0$)
 $\dot{n}_0 = \dot{n}_1$

$\text{so } f_A = \frac{2}{3} = 0.667$

b) $r_A = -kP_A + \frac{k}{2}P_B$

$$\rightarrow y_A P = P_A \quad y_B P = P_B$$

$$\text{so, } r_A = kP \left(\frac{1-y_A}{2} - y_A \right) \quad \text{(+1)}$$

$$y_B + y_A = 1 \rightarrow y_B = 1 - y_A \quad \text{(+1)}$$

c) $\frac{df_A}{dV} = -\frac{r_A}{\dot{n}_1} = -\frac{kP}{2\dot{n}_1} (1-3y_A)$ ② need to relate y_A to f_A : $y_A = \frac{\dot{n}_{A,0}}{\dot{n}_1}$

↳ # moles does not change in reactor

$$\dot{n}_{A,0} = \dot{n}_1 - \dot{n}_1 f_A = \dot{n}_1 (1-f_A), \text{ so } \underline{y_A = 1-f_A} \rightarrow \text{plug into design eqn.} \quad \text{(+2)}$$

$$\frac{df_A}{dV} = -\frac{kP}{2\dot{n}_1} (1-3(1-f_A)) = -\frac{kP}{2\dot{n}_1} (3f_A - 2) = \frac{kP}{\dot{n}_1} \left(1 - \frac{3}{2}f_A \right)$$

Rearrange: $\int \frac{df_A}{(1-\frac{3}{2}f_A)} = \frac{kP}{\dot{n}_1} \int dV \rightarrow -\frac{2}{3} \ln \left(1 - \frac{3}{2}f_A \right) = \frac{kP}{\dot{n}_1} V + C$ ①

@ $V=0, f_A=0 \rightarrow C=0$, so $\ln \left(1 - \frac{3}{2}f_A \right) = -\frac{3}{2} \frac{kP}{\dot{n}_1} V$ ①

$V = -\frac{2\dot{n}_1}{3kP} \ln \left(1 - \frac{3}{2}f_A \right)$

$f_A = \frac{2}{3} - \frac{2}{3} \exp \left(-\frac{3}{2} \frac{kP}{\dot{n}_1} V \right)$

d. (version 3) V needed to achieve $f_A = 0.25$ w/n₁ = 100 mol/s, k = 0.01 mol/s Latm, P = 10 atm

$$V = -\frac{2}{3} \frac{n_1}{kP} \ln \left(1 - \frac{3}{2} f_n \right) = \boxed{313} \quad L = V$$

$$d. (\text{version } 4) \quad f_A = \frac{2}{3} - \frac{2}{3} \exp\left(-\frac{3}{2} \frac{kP}{n_1} V\right) \quad V=500L, \text{ all else same above}$$

$f_A = 0.352$

(42)

e, f, g all same as version 1&2:

$$e. \dot{n}_2 = \dot{n}_3 = 2\dot{n}_1, \quad f. \dot{n}_{2,A} = \frac{2\dot{n}_1}{(1+f_A)} \quad g. Y_{A,3} = \frac{(1-f_A)}{(1+f_A)}$$

$$h. \frac{df_A}{dV} = -\frac{r_A}{n_{2A}} \quad (1) \quad r_A = \frac{kP}{2}(1-3y_A), \text{ so } \frac{df_A}{dV} = -\frac{kP}{4n_1} \left(1-3\left(\frac{1-f_A}{1+f_A}\right)\right)(1+f_A)$$

$$\frac{df_A}{dV} = -\frac{kP}{4n_1} \left(1 + f_A - 3 + 3f_A \right) = -\frac{kP}{4n_1} (4f_A - 2)$$

(+5) $= -\frac{kP}{2n_1} (2f_A - 1)$

$$\text{Rearrange: } \frac{df_A}{1-2f_A} = \frac{kP}{2n_1} dV \Rightarrow -\frac{1}{2} \ln(1-2f_A) = \frac{kP}{2n_1} V + C$$

$$\text{At } V=0, f_A=0, \text{ so } C=0 \quad (1)$$

$$V = -\frac{n_1}{kP} \ln(1-2f_A) \quad (2) \quad f_A = \frac{1}{2} - \frac{1}{2} \exp\left(-\frac{kPV}{n_1}\right)$$

i) Same as Version 1&2:

$$F_A = \frac{2f_A}{1+f_A}$$

$$\text{i) } F_A = \frac{n_1 - n_{5A}}{n_1} ?$$

$n_{3A} = 2n_{5A}$ from split balance $\textcircled{+1}$

$$n_{3A} = n_{2A}(1-f_A) \text{ and } n_{2A} = \frac{2n_1}{1+f_A} \text{ so } n_{3A} = 2n_1 \frac{(1-f_A)}{(1+f_A)} = 2n_{5A}$$

$$n_{5A} = n_1 \frac{(1-f_A)}{(1+f_A)} \textcircled{+2}$$

$$\therefore F_A = \frac{n_1}{n_1} \left[\frac{1 - \frac{(1-f_A)}{(1+f_A)}}{1} \right] = 1 - Y_{A,3}$$

$$\textcircled{+2} \quad F_A = 1 - \frac{(1-f_A)}{(1+f_A)} = \frac{1+f_A - 1+f_A}{1+f_A} = \boxed{\frac{2f_A}{1+f_A} = F_A}$$