University of California, Berkeley

ME106

Midterm 1

1) Consider the bathroom scale on the left side of Fig. 1. It registers weight $|W|$ upon it. Whatever is on the scale has a force $|W|\,\hat{\mathbf{z}}$ exerted on it by the scale. If the only thing on the scale is the atmosphere, then $|W| = A\,P_{atm}$, where A is the cross-sectional area of the scale, P_{atm} is the pressure of the atmosphere (which we shall assume is independent of z for this problem), and the gravity g is in the $-z$ direction. Most scales can be adjusted so that when only the atmosphere is present, $W = 0$, that is, the scale has been offset to measure the gage pressure. Let's assume that our scale has not been offset, and the weight registered by the scale includes the weight of the atmosphere. Now stand on this scale and find your weight as shown on the right side of Fig. 1. This is more complex than it sounds, so we shall approximate your body as a cuboid with horizontal cross-sectional area A, and we shall approximate your two feet – each with horizontal cross-sectional a, as in the figure. Your feet are infinitesimally high so that there is a thin film of air at pressure P_{atm} between the scale and the bottom of your body.(The figure shows a layer of air between the top of the scale and the bottom of your body. It is exaggerated, so it is not infinesimally thick.) We shall assume that your feet both make pefect conact (a seal!) with the top of the scale, so that there is no air between the bottoms of your feet and the scale. The combined mass of your body and two feet is M. Note that the horizontal cross section of the thin layer of air between the scale and your body is $(A - 2a)$.

What weight $|W|$ does the scale register on the right side of Fig. 1? In this picture, the flatter your feet are the bigger is a. If you want to have the scale measure a low weight, do you want a to be large or small? Does your answer have the correct dimensions?

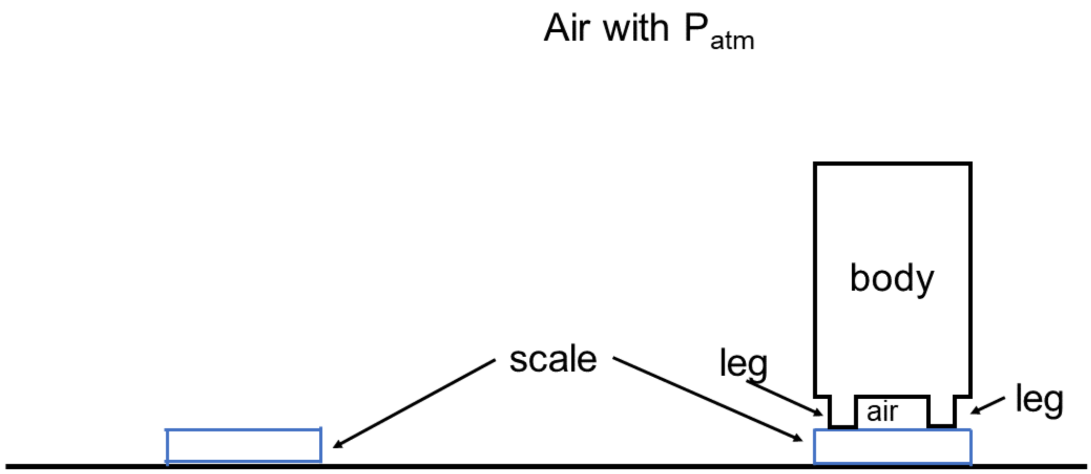

Figure 1: The scale on the left is weighing the air with pressure P_{atm}. On the right, the scale weighs some air along with your body and feet. Your feet make a "seal" with the scale. The cross-sectional area of the bottoms of your two feet sum to $2a$.

2) Consider the multi-density fluid that we examined in Lecture 3, Fig, 3, which we repeat here.

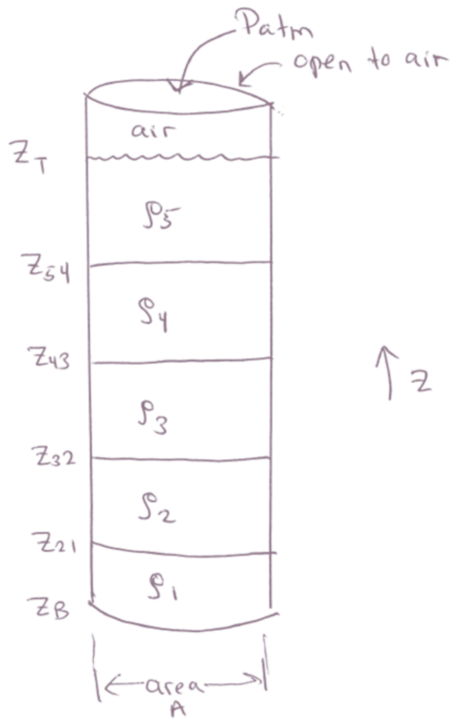

Figure 2:

Let the fluid be contained in a cylinder of radius R. Fig. 3 shows the cylinder, which is surrounded by air at constant pressure P_{atm} as viewed from above.

The cylinder is made of six sections (of equal area), and each extends from the bottom at $z = z_B$ to the top of the cylinder shown in Fig. 2. The six sections are sealed together with glue. Do we need strong glue? In particular, what is the force **F** that the glue provides to hold the section indicated with the red arrow (in Fig. 3) in place? This **F** balances the pressure force that the fluids (including air) exert on the section so that the section is in

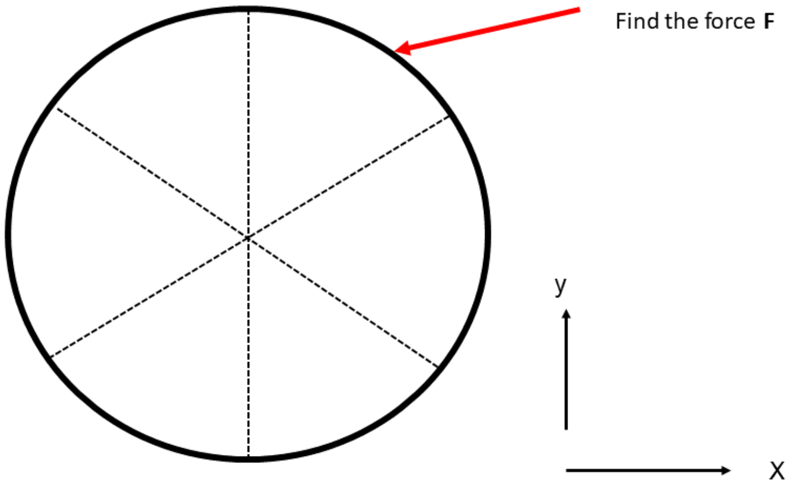

Figure 3: Each of the six cylindrical sections of the tube encasing the fluid spans $60°$. The x and y directions are shown with black arrows. You are to calculate the force **F** that the glue provides to hold the section indicated by the red arrow in place. **F** balances the fluids' pressure force on the section such that the total force on the section is equal to zero.

steady equilibrium. Remember that the force is a vector, so make certain that your answer is a vector. Does your answer have the correct dimensions? You can use the information in Lecture 3 if you think it would be helpful. $sin\ 30° = 1/2$ and $cos\ 30° = \sqrt{3}/2$.

3) Consider the rectangular tank in Fig, 4 that contains four liquids with different mass densities: ρ_1, ρ_2, ρ_3, and ρ_4, as well as air at pressure P_{atm}. Gravity g is in the $-z$ direction. The tank has length L in the y-direction (into the paper). The upper partition in the tank goes from the point **1** straight upward and continues above the surfaces of the liquids. The lower partition goes from the point **2** (which is directly beneath point **1**) to the bottom of the tank at $x = 0$ and $z = z_B$. The middle part of the partition is bent into a right angle as shown. The longer part of the middle partition has length $\sqrt{3}/2\,H$ and the shorter side has length $H/2$. The horizontal interfaces of the fluids are at $z = 0$, $z = z_4$, and $z = z_1$. The location of the $90°$ bend in the middle partition is at $z = z_1$. The middle partition has no mass, and the three partitions cover the entire y extent of the tank so that the liquids on the left and right sides of the tank do not mix. There are fasteners at points **1** and **2** that together exert a force **F** on the middle partition. **F** is such that the all of the forces (including those to the pressures) sum to zero. Find **F**.

Hint: If you decide to use Archimedes' rule to simplify your calculation, and if you do a "thought experiment" to find the fluid pressure force on a surface, choose a control volume and choose the fluid(s) that surround that volume in the "thought experiment" such that: (1) you are able to calculate (and then use) the pressure that is in hydrostatic equilibrium with the fluid(s) in the "thought experiment", and *not* the fluid(s) in the actual problem; (2) make your choices such that the fluid forces on the surface that you are interested in finding have the same hydrostatic pressure acting upon it in both the "thought experiment" and the actual problem you are trying to solve; (3) remember that the surface you are interested in has *two* sides, so make sure that your calculation in the "thought experiment" deals with the pressure forces on the side that is relevant to the problem you need to solve; (4) when applying Archimedes' rule, the fluid(s) displaced by the volume in the "thought experiment"

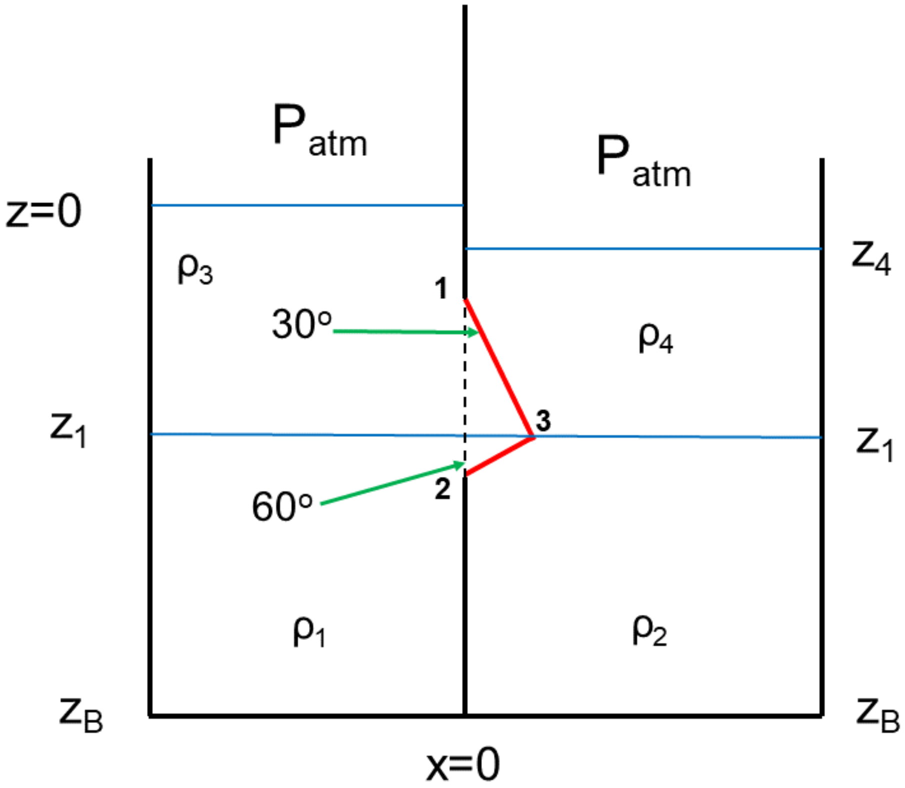

Figure 4: Rectangular tank with upper, lower, and middle partitions in thick black lines. You are to find the combined force F that the upper and lower partitions exert at points **1** and **2** on the middle partition such that the forces on the middle partition (shown in red) sum to zero. The middle partition is bent at location **3** into a right angle. The vertical broken line at $x = 0$ is not a partition but is for reference. The long upper part of the middle partition in red makes an angle of an $30°$ with respect to the vertical line. The short lower part of the middle partition makes an angle of $60°$ with respect to the vertical line. The interface between the air and the liquids is shown with horizontal blue lines. The interfaces between the liquids of different densities are sown with blue horizontal lines. Note that the (x, z) locations of points **1**, **2**, and **3** are $(0, z_1 + 3H/4)$, $(0, z_1 - H/4)$, $(\sqrt{3}H/4, 0)$, respectively.

is not necessarily the fluid(s) in the actual problem, but rather, is the fluid(s) displaced by the volume in the "thought experiment" which has densities that are consistent with the hydrostatic pressure used in the "thought experiment".