Name:_

SID: _

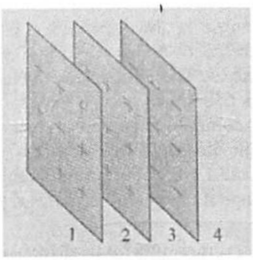
Physics 7B Midterm 2 – Fall 2019
Professor A. Lanzara

TOTAL POINTS: 100

Show all work, and take particular care to explain what you are doing. Partial credit is given. Please use the symbols described in the problems. Define any new symbol that you introduce and label any drawings that you make. All answers should be in terms of given variables. If you get stuck, skip to the next problem and return later.

PROBLEM 1 (15 pts.)

a) **(5 pts.)** The figure below shows sections of three infinite flat sheets of charge, each carrying surface charge density with the same magnitude s. Find the magnitude and direction of the electric field in each of the four regions shown.



b) **(5 pts.)** A proton (mass m_p and charge q) with kinetic energy K is fired perpendicular to the face of a large charged sheet. The sheet has a uniform surface charge density σ.

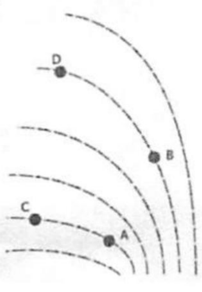
i. What is the magnitude of the force on the proton?

ii. At what distance should the proton be fired so that it stops right at the surface of the plate?

c) **(5 pts.)** Consider the figure below. The dashed lines are the equipotential surfaces of some charged object ~~and the solid lines with arrows are its electric field lines.~~

i. Let W_{AB} be the magnitude of the work required to move a particle of charge q from point A to point B. Similarly, let W_{CD} and W_{AC} be the magnitudes of the work needed to move the same particle from C to D and A to C, respectively. Rank these values in order of largest to smallest.

ii. Let E_A, E_B, E_C, and E_D be the electric field magnitudes at the four points. Rank them from largest to smallest.



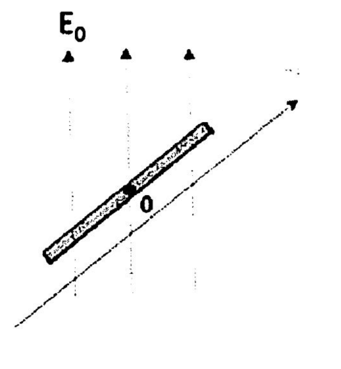
PROBLEM 2 (20 pts.)

Consider a thin bar of length 2L. Initially it lies along the x-axis. The linear charge density of the bar is λ, where $\lambda = ax$ (a>0) and x is the longitudinal coordinate with respect to the center of the bar. The bar is pivoted at its center, which is located at the origin. Thus it can rotate, but not move.

a) **(15 pts.)** Calculate the electric potential at a point P that lies a distance d directly above the end of the positive part of the rod.

A constant, uniform and horizontal electric field E_0 is turned on (see the figure below).

b) **(5 pts.)** Describe what will happen to the bar. At which angle (relative to the direction of the electric field) will the bar reach equilibrium and stop rotating?



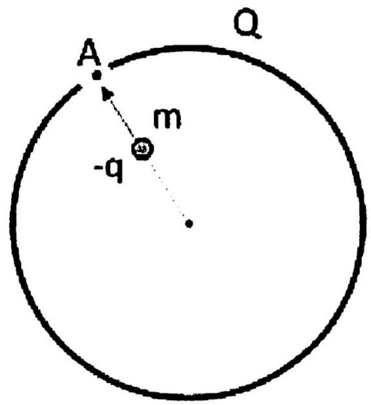
PROBLEM 3 (20 pts.)
Consider a spherical conducting shell of radius R. The sphere is uniformly charged, with total charge Q.

a) **(5 pts.)** Find the electric field at a point just above the surface of the sphere.

A circular aperture in now cut out of the sphere and point A lies at the center of this hole. A particle of charge -q and mass m is placed at the center of the cut sphere and can escape through the aperture. The particle is thrown from the center of the sphere and escapes radially.

b) **(10 pts.)** Find the electric field at point A

c) **(5 pts.)** Find the minimum velocity v_0 that the particle needs to have in order to escape from the attraction of the sphere



PROBLEM 4 (22 pts.)

A cylindrical capacitor consists of a cylinder of radius a surrounded by a coaxial cylindrical shell of inner radius b. Both cylinders have length h which is much greater than the separation of the cylinders (b-a), so we can neglect edge effects. The capacitor is charged with charge Q on the inner cylinder and charge -Q on the outer cylinder.

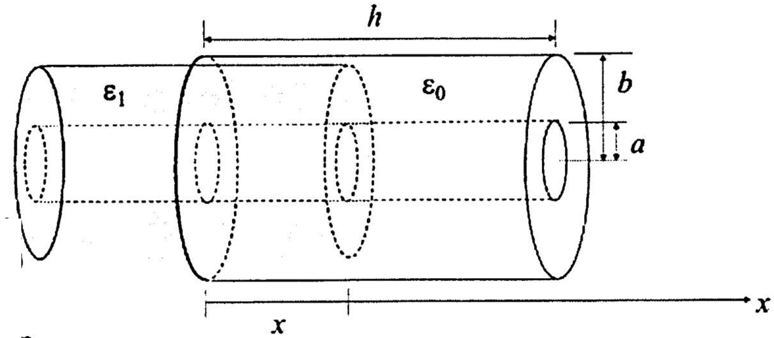
a) **(7 pts.)** Determine a formula for the capacitance.

b) **(4 pts.)** Find the total energy U that can be stored in the capacitor.

After some time the space between the cylinders is filled with a homogeneous dielectric of dielectric constant ε_1. See the figure below.

c) **(5 pts.)** What is the equivalent capacitance of the resulting system?

d) **(6 pts.)** Find the expression of the force that is pulling the dielectric in.



PROBLEM 5 (23 pts.)

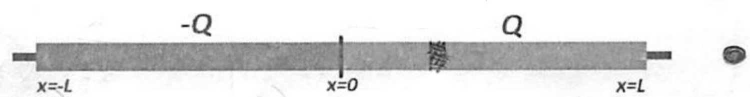
Two identical rods of length L lie on the x-axis and carry uniform charges +Q and -Q, as shown below.

a) **(12 pts.)** Find an expression for the electric field strength as a function of position x for points to the right of the right-hand rod

b) **(5 pts.)** Show that your result has the $1/x^3$ dependence of a dipole field for $x >> L$.

c) **(3 pts.)** What is the dipole moment of this configuration?

d) **(3 pts.)** How would the behavior of the field change in the limit $x >> L$ if the two rods were replaced by a single rod of length $2L$ and charge +Q?



Physics 7B Midterm 2 Equation Sheet Fall 2019

For a point charge Q:

$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}\hat{r}$ $\qquad$ $V = \frac{1}{4\pi\epsilon_0}\frac{Q}{r}$

$C = \frac{\epsilon_0 A}{d}$ $\qquad$ $F = -\frac{dU}{dx}$

$K = \frac{1}{2}mv^2$ $\qquad$ $\vec{p} = Q\vec{d}$

Far from a dipole: $V = \frac{1}{4\pi\epsilon_0}\frac{p\cos(\theta)}{r^2}$

Spherical Coordinates: $dV = r^2\sin(\theta)\, dr d\phi d\theta$

Cylindrical Coordinates: $dV = \rho\, d\rho d\phi dz$

Polar Coordinates: $dA = r\, dr d\theta$

$$\int \frac{1}{(a^2+x^2)^{1/2}}dx = \ln(x+\sqrt{a^2+x^2}) + C$$

$$\int \frac{1}{(a^2+x^2)}dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$$

$$\int \frac{x}{(a^2+x^2)^{1/2}}dx = \sqrt{a^2+x^2} + C$$

$$\int \frac{1}{(a^2+x^2)^{3/2}}dx = \frac{x}{a^2\sqrt{a^2+x^2}} + C$$

$$\int \frac{1}{(a^2+x^2)^2}dx = \frac{1}{2a^3}\left(\frac{ax}{a^2+x^2} + \tan^{-1}\left(\frac{x}{a}\right)\right) + C$$

$$\int \frac{x}{(a^2+x^2)^{3/2}}dx = -\frac{1}{\sqrt{a^2+x^2}} + C$$

$$\int \frac{1}{(a+x)^2}dx = -\frac{1}{a+x} + C$$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2}x^2 + \cdots$$

$$\sin(x) = x - \frac{x^3}{6} + \cdots$$

$$e^x = 1 + x + \frac{x^2}{2} + \cdots$$

$$\cos(x) = 1 - \frac{x^2}{2} + \frac{x^4}{24} + \cdots$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} + \cdots$$

Spherical Coordinates

If (r, θ, ϕ) are the spherical coordinates and (x, y, z) the rectangular coordinates of a point P, then

$$
\begin{aligned}
x &= r\sin(\theta)\cos(\phi) \\
y &= r\sin(\theta)\sin(\phi) \\
z &= r\cos(\theta)
\end{aligned}
$$

Recall that the volume differential in spherical coordinates can be written as

$$
\begin{aligned}
dV &= r^2\, dr d\Omega = r^2 \sin(\theta)\, dr d\phi d\theta \\
d\Omega &= \sin(\theta) d\theta d\phi
\end{aligned}
$$