Problem 1 – Short Answers (20 points)

[2 pts.] (a) 

\[ 10V \pm 300 \Omega \rightarrow V_1 \]

\[ 100 \Omega \]

\[ V_1 = 2.5V \]

[2 pts.] (b) 

\[ 10A \uparrow \rightarrow \]

\[ 4 \Omega \quad i_1 \]

\[ 6 \Omega \]

\[ i_1 = 6A \]

[2 pts.] (c) 

\[ 20V \pm \]

\[ 1 \rightarrow \]

\[ 3 \Omega \quad 3 \Omega \quad 3 \Omega \]

\[ 2 \Omega \]

\[ i = 5A \]

[4 pts.] (d) 

For what value of \( R_L \) is power in \( R_L \) maximized?

\[ R_L = 2 \omega \]

What is the maximum power dissipated in \( R_L \)?

\[ P = \frac{10V}{4\omega} \times \frac{10V}{4\omega} R \]

\[ P = \frac{100}{16} \times 2 = 12.5 \text{ watts} \]
Problem 1 (cont.)

[6 pts.] (e) \[1\mu F = C_1, 0.5\mu F = C_2, 0.2\mu F = C_3\]

\[160 V \pm \]

\[
\begin{align*}
q_1 &= q_2 = q_3 \\
C_1 V_1 &= C_2 V_2 = C_3 V_3 \\
V_1 + V_2 + V_3 &= 160 V
\end{align*}
\]

Determine charge \((q)\) and voltage across each capacitor.

<table>
<thead>
<tr>
<th>Capacitor</th>
<th>Charge ((q))</th>
<th>Voltage ((V))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(q_1)</td>
<td>(2 \times 10^{-5} C)</td>
<td>(20 V)</td>
</tr>
<tr>
<td>(q_2)</td>
<td>(2 \times 10^{-5} C)</td>
<td>(40 V)</td>
</tr>
<tr>
<td>(q_3)</td>
<td>(2 \times 10^{-5} C)</td>
<td>(100 V)</td>
</tr>
</tbody>
</table>

[4 pts.] (f) Short Question

Here is a circuit fragment – a 100-ohm resistor imbedded in a very large circuit that provides the currents shown here. Find the unknown current \(I_x\) and the voltage \(V_{ab}\) across the resistor.

![Circuit Diagram]

\[I_x = \boxed{\text{unknown}}\]

\[V_{ab} = \boxed{\text{unknown}}\]

\[10 mA - 30 mA - (-40 mA) + I_x = 0\]

\[I_x = -20 mA\]

\[V_{ab} = -100 \cdot 20 mA = -2 V\]
Problem 2 (20 points)

(a) Write (but do not solve) a minimum set of equations that could be solved for unknowns $V_2$ and $V_3$ using Kirchhoff’s Current Law, KCL. (Equations in box for full credit.)

\[
\begin{align*}
\alpha & \quad V_2 : \quad \frac{V_0 - V_2}{R_1} - \frac{(V_2 - V_3)}{R_2} = 0 \\
\alpha & \quad V_3 : \quad \frac{V_2 - V_3}{R_2} - \frac{V_3}{R_3} - \beta \frac{V_3}{R_3} = 0
\end{align*}
\]

(b) Write (but do not solve) a minimum set of equations that could be solved for unknowns $i_1, i_2$ using Kirchhoff’s Voltage Law (KVL). (Equations in box for full credit.)

\[
\begin{align*}
\nu & \quad V_0 - i_1 R_1 - i_1 R_2 - i_2 R_3 = 0 \quad (\text{kVL around loop}) \\
\nu & \quad i_1 - i_2 - \beta i_2 = 0 \quad (\text{kCL at node 2})
\end{align*}
\]
Problem 3 – Equivalent Circuits (20 points)

[2 pts.] (a) Draw the Thévenin equivalent circuit.

\[
\text{\begin{align*}
V_{TH} = 10.25 V \\
R_{TH} = 10 \Omega
\end{align*}}
\]

[14 pts.] (b) \[R_{TH} = \frac{1}{10} \]

\[
V_{TH} = 102.5 V
\]

[2 pts.] (c) Draw the Norton equivalent circuit.

[2 pts.] (d) In general, if you know \( V_{TH} \) and \( R_{TH} \), how do you determine \( R_N \) and \( I_N \)?

\[
R_N = R_{TH} \quad I_N = \frac{V_{TH}}{R_{TH}}
\]
Problem 4 – Lab Related Question (18 points)

Suppose you buy a strange-looking battery at the Berkeley Surplus Center and want to find its Thévenin equivalent circuit experimentally. You have a multimeter with voltage, current and resistance scales, and you also have one resistor $R$, one capacitor $C$, and one inductor $L$. Incidentally, you don’t want to short circuit the battery – it would be bad for it!

a) How can you find $V_{Th}$ safely?

b) How can you find $R_{Th}$ safely?

Draw the circuits showing how you’d connect the meter, the battery and any other components, and write any equations you will use in the process.

\[ V_{RAT} = V_{TH} \]

a) Open circuit voltage $V_{RAT} = V_{TH}$, so just use multimeter voltage scale.

\[ \text{voltage meter} \]

b) We know $R_{TH} = \frac{V_{TH}}{I_{sc}}$, but we don’t want to measure short circuit current. We can use known $R$ and multimeter to find $R_{TH}$:

\[ V_m = V_{TH} \cdot \frac{R}{R + R_{TH}} \]

\[ R_{TH} = R \left( \frac{V_{TH}}{V_m} - 1 \right) \]

[if you use a capacitor you instantaneously short $V_{RAT}$]

Could use

\[ R_{TH} = \frac{1}{C} \frac{1}{RC} - R \]
Problem 5 – RL Circuit

An RL circuit with a voltage source and a very fast-acting switch is shown. the values of the components are: $V_s = 2 \text{ V}; R_1 = 20\Omega; R_2 = 980\Omega; L = 0.1 \text{ H}$ At time $t = 0$, a long time after the switch has closed, the switch opens.

[2 pts.] (a) What is the current $i_L$ that is flowing at time $t = -2s$?

$$\frac{V_s}{R_1} = \frac{2}{20} = 0.1 \text{ amp}$$

[2 pts.] (b) What is the time constant for the RL circuit?

$$\tau = \frac{L}{R_1+R_2} = \frac{0.1}{20+980} = 10^{-4} \text{ sec}$$

[6 pts.] (c) Write KVL for time $t = 0^+$.

for inductor, current can not change instantaneously. $V_L = L \frac{di}{dt}$

$$-i_L R_2 - 4 R_1 - V_L = 0$$

[4 pts.] (d) Find the voltage $V_a(t = 0^+)$.

$$-i_L R_2 = -(0.1)980 = -98\text{ V}$$

[8 pts.] (e) Determine $v_a(t)$ for $t > 0$.

$$v_a(t) = v_a(0^+) \left[1 - \frac{t}{\tau}\right] e^{-t/\tau}$$

$$\lim_{t \to \infty} v_a(t) = 0$$

$$v_a(0^+) = -98e^{-t/10^{-4} \text{ sec}^{-1}}$$