Name: $\qquad$

SID: $\qquad$

## Physics 7B Midterm 1 - Fall 2020 <br> Professor R. Birgeneau

## Total Points: $100+5$ ( Problems)

This exam is out of 100 points with 5 bonus points. Show all your work and take particular care to explain your steps. Partial credit will be given. Use symbols defined in problems and define any new symbols you introduce. If a problem requires that you obtain a numerical result, first write a symbolic answer and then plug in numbers. Label any drawings you make. Good luck!

## Problem 1 (20 pts.)

A mass $M=2 \mathrm{~kg}$ of ice at temperature $T=-40^{\circ} \mathrm{C}$ is in a thermally isolated container. 4 kg of water at temperature $T=90^{\circ} \mathrm{C}$ is added to the container. The pressure $P$ of the water-ice system is 1 atm .

Assume that the specific heat of the water does not depend on the temperature and is $c_{w}=1$ $\mathrm{cal} /(\mathrm{g} \mathrm{K})$. The latent heat of fusion of ice is $L=80 \mathrm{cal} / \mathrm{g}$ and the heat capacity ice is $c_{i}=0.5 \mathrm{cal} /(\mathrm{g}$ $\mathrm{K})$. Neglect the change of volume of ice and water and the specific heat of the container.
(a) (10 pts.) What is the final equilibrium temperature?
(b) (10 pts.) What is the approximate change in entropy of the water-ice system between the initial and final state?

## Problem 2 (20 pts.)

Consider an ideal gas of $N$ molecules contained in a cubic room with sides of length $L$ at temperature $T$ and pressure $P$
(a) (6 pts.) Take the average $x$ component of a molecule's velocity to be $\bar{v}_{x}$. Using $\bar{v}_{x}$ and the quantities above, derive an expression for the frequency $f$ with which gas molecules strikes a wall. You may also use fundamental constants.
(b) (6 pts.) Show that the frequency can be rewritten as

$$
\begin{equation*}
f \approx \frac{P L^{2}}{\sqrt{4 m k T}} \tag{1}
\end{equation*}
$$

where $m$ is the mass of the molecule
(c) (8 pts.) Assume a cubic air-filled room is at sea level, has a temperature $20^{\circ} \mathrm{C}$, and has sides of length $L=3 \mathrm{~m}$. Determine $f$. Assume that air is $80 \%$ Nitrogren and $20 \%$ Oxygen.

## Problem 3 (20 pts.)

(a) ( 6 pts.) Estimate the rate at which the whole Earth receives energy from the Sun. Recall that the solar constant is $1350 \mathrm{~W} / \mathrm{m}^{2}$ and that the radius of the Earth is $R_{\text {Earth }}=6 \times 10^{6} \mathrm{~m}$.
(b) ( 6 pts.) The Earth also radiates energy into space. Assume that the Earth is in equilibrium and it ratiates as much heat into space as it absorbs from the sun. Also assume the Earth is a perfect emitter and determine its surface temperature.
(c) (8 pts.) Consider a 100 W lightbulb that generates heat at a rate of 95 W . The heat is dissipated through a glass bulb that has a radius of 2.0 cm and is 0.4 mm thick. Determine the difference in temperature between the inner and outer surfaces of the glass bulb. The thermal conductivity of glass is $0.84 \mathrm{~J} /\left(\mathrm{s} \cdot \mathrm{m} \cdot \mathrm{C}^{\circ}\right)$.

## Problem 4 (20 pts.)

If we consider using the van der Waals equation of state for oxygen gas, experiments find that $a=0.14 \mathrm{~N} \cdot \mathrm{~m}^{4} / \mathrm{mol}^{2}$ and $b=3.2 \times 10^{-5} \mathrm{~m}^{3} / \mathrm{mol}$ yield the best fits. Determine the pressure in 1.0 mol of the gas at $0^{\circ} \mathrm{C}$ if its volume is 0.70 L , calculated using
(a) (6 pts.) the van der Waals equation
(b) ( 6 pts.) the ideal gas law

Now consider a 0.5 mol sample of $\mathrm{O}_{2}$ gas that is in a large cylinder with a movable piston on one end so it can be compressed. The initial volume is large enough that there is not a significant difference between the pressure given by the ideal gas law and that given by the van der Waals equation.
(c) (8 pts.) As the gas is slowly compressed at constant temperature (300 K), at what pressure does the van der Waals equation give a volume that is $5 \%$ different than the ideal gas law volume? Use the values of $a$ and $b$ given above.

## Problem 5 (20 pts.)

Depicted below is a cycle that consists of an isobaric path, an isovolumetric path, and two adiabatic paths in the PV plane. The working substance is $n$ moles of a diatomic gas. Assume the temperatures of the four points to be $T_{a}, T_{b}, T_{c}, T_{d}$ and the volumes to be $V_{a}, V_{b}, V_{c}, V_{d}$.
(a) (5 pts.) Express $Q_{L}$ in terms of the given quantities and fundamental constants.
(b) ( 5 pts .) Express $Q_{H}$ in terms of the given quantities and fundamental constants.
(c) (10 pts.) Derive an expression for the efficiency in terms of only the volumes and fundamental constants.


## Bonus (5 pts.)

At room temperature an ideal gas of $\mathrm{CO}_{2}$ has a molar specific heat, $C_{p}$, in units of $R$, of approximately 4.5. Explain quantitatively the origin of this specific heat.

