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## Section:

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## Physics 7B Lecture 3 Final - Fall 2018 <br> Professor A. Lanzara

This exam is out of 100 points. Show all your work and take particular care to explain your steps. Partial credit will be given. Use symbols defined in problems and define any new symbols you introduce. If a problem requires that you obtain a numerical result, first write a symbolic answer and then plug in numbers. Label any drawings you make. Good luck!

Problem 1 ( 15 pts.) Consider a Carnot engine operating between temperatures $\mathrm{T}_{h}$ and $\mathrm{T}_{c}$, where $\mathrm{T}_{c}$ is above the ambient temperature $\mathrm{T}_{0}$. A second Carnot engine is set up that operates between $T_{c}$ and the ambient temperature $T_{0}$, as shown in the figure below. The heat output of the first engine acts as the asborbed heat of the second engine.
a) Draw the P-V diagram for the two-stage engine
b) Draw the P-V diagram for the equivalent engine
c) Calculate the efficiency of the equivalent engine. What is the role of $T_{c}$ ?


Problem 2 ( 10 pts.) A pan contains 500 g of water at $20^{\circ} \mathrm{C}$. What heat flow does it require to raise the temperature of the water to $100{ }^{\circ} \mathrm{C}$ and vaporize it at 1 atm ?Take the specific heat of water to be $\mathrm{c}_{w}=4000 \mathrm{~J} / \mathrm{kg} . \mathrm{C}^{\circ}$ and the latent heat of vaporization to be $L_{w}=2200 \mathrm{~kJ} / \mathrm{kg}$.

## Proble 3 (15 pts.)

a) The figure below depicts a tetrahedron immersed in a uniform electric field with magnitude $E$. Assume that each face is an equilateral triangle with edge length $L$.
(i) What is the total flux through the tetrahedron?
(ii) Calculate the electric flux through the top three faces.

b) A constant current flows through a conical conductor as shown below. End surfaces $S_{1}$ and $S_{2}$ are equipotential surfaces that are not at the same potential. The metal has a uniform conductivity $\sigma$. Assume the electric field is confined to the interior of the conductor.
(i) Through which plane does the greatest current flow?
(ii) Through which plane is the greatest electric flux?
(iii) How does the magnitude of the electric field $E$ vary along the central axis moving from $S_{1}$ to $S_{2}$ ?


Problem 4 ( $\mathbf{1 5}$ pts.) For all the capacitors and dielectric configurations below, assume that the top plate has a charge +Q and the bottom plate has a charge -Q . Take the separation between the plates to be $d$ and the area of the plates to be $A$.

(a) Calculate the effective capacitance of the capacitor above. The space between the capacitor plates is completely filled with two blocks of dielectrics with dielectric constants $K_{1}$ and $K_{2}$.
(b) Repeat part (a) for the new capacitor below, which also is filled with two dielectrics with dielectric constants $K_{1}$ and $K_{2}$. Note that $d_{1}+d_{2}=d$.


Problem 5 ( 15 pts.) A wire carrying a current $I$ splits into two channels of resistance $R_{1}$ and $R_{2}$, respectively, forming a circuit. The loop is fixed and cannot be translated, but is free to rotate. The wire enters the space between the two poles of a magnetic with a uniform magnetic field that runs from one pole piece to the other, as shown below. What is the torque on the circuit about the wire axis, given that the wires are a distance $d$ apart and that the length of the split is $L$ ?


Problem 6 (20 pts.) A wire is bent into a circular arc of radius $r$, as shown below. An additional straight length of wire, OP, is free to pivot about O and makes sliding contact with the arc at P. Finally, another straight length of wire, OQ, completes the circuit. All the wires have cross-sectional area $A$ and resistivity $\rho$. The entire arangement is located in a magnetic field $B$ that is directed out of the plane of the figure.
(a) Find the resistance of the loop OPQO as a function of $\theta$.
(b) Find the magnetic flux through the loop as a function of $\theta$. Hint: Think of the area for certain values of $\theta$.

We now consier the case where the straight wire OP starts from rest at $\theta=0$ and has a constant angular acceleration of $\alpha$.
(a) Find the induced emf in the loop as a function of time in terms of $\alpha, B$, and other given constants.
(b) For what value of $\theta(t)$ is the induced current in the loop a maximum?


## Problem 7 (10 pts.)

a) Consider two inductors $L_{1}$ and $L_{2}$ that are in series. Derive the equivalent inducatance.
b) Consider two inductors $L_{1}$ and $L_{2}$ that are in parallel. Derive the equivalent inductance.

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\begin{aligned}
& \text { Thermodynamics } \\
& \text { and Mechanics } \\
& \Delta l=\alpha l_{0} \Delta T \\
& \frac{d Q}{d t}=-k A \frac{d T}{d x} \\
& d S=\frac{d Q}{T} \\
& \Delta S_{\text {syst }}+\Delta S_{e n v}>0 \\
& \oint d S=0 \\
& \Delta V=\beta V_{0} \Delta T \\
& e=\frac{W_{n e t}}{Q_{i n}} \\
& C_{p}-C_{V}=R=N_{A} k_{B} \\
& \oint \vec{E} \cdot d \vec{l}=-\frac{d \Phi_{B}}{d t} \\
& \vec{B}=\frac{\mu_{0}}{4 \pi} \int \frac{I d \vec{l} \times \hat{r}}{r^{2}} \\
& \vec{\mu}=N I \vec{A} \\
& \mathrm{COP}_{\text {ideal }}=\frac{T_{L}}{T_{H}-T_{L}} \\
& \left|\vec{F}_{\text {cent }}\right|=\frac{m v^{2}}{r} \\
& v=\frac{d x}{d t} \\
& a=\frac{d v}{d t} \\
& \omega=\frac{d \theta}{d t} \\
& \alpha=\frac{d \omega}{d t} \\
& \text { Electromagnetism } \\
& \oiint \vec{E} \cdot d \vec{A}=\frac{Q_{\text {encl }}}{\epsilon_{0}} \\
& \text { Constants and Formulas } \\
& k_{B}=1.38 \times 10^{-23} \mathrm{~J} / \mathrm{K} \\
& C=K C_{0} \\
& P=I V \\
& q=1.6 \times 10^{-19} \mathrm{C} \sim 10^{-19} \mathrm{C} \\
& \frac{1}{4 \pi \epsilon_{0}} \sim 10^{10} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{C}^{2} \\
& R=\frac{\rho l}{A} \\
& \Delta V=-\int \vec{E} \cdot d \vec{l} \\
& \sqrt{3} \sim 1.7 \\
& \sqrt{2} \sim 1.4 \\
& \vec{E}=-\nabla V \\
& \oiint \vec{B} \cdot d \vec{A}=0 \\
& \ln (2) \sim 0.69 \\
& \ln (3) \sim 1.09 \\
& \cos (x)=1-\frac{x^{2}}{2}+\frac{x^{4}}{24} \cdots \\
& \sin (x)=x-\frac{x^{3}}{6}+\cdots \\
& e^{x}=1+x+\frac{x^{2}}{2}+\cdots \\
& \ln (1+x)=x-\frac{x^{2}}{2}+\frac{x^{3}}{3}+\cdots \\
& \int \frac{1}{\left(a^{2}+x^{2}\right)^{1 / 2}} d x=\ln \left(x+\sqrt{a^{2}+x^{2}}\right)+C \\
& \int \frac{1}{\left(a^{2}+x^{2}\right)^{3 / 2}} d x=\frac{x}{a^{2} \sqrt{a^{2}+x^{2}}}+C \\
& \int \frac{x}{\left(a^{2}+x^{2}\right)^{1 / 2}} d x=\sqrt{a^{2}+x^{2}}+C \\
& \int \frac{x}{\left(a^{2}+x^{2}\right)^{3 / 2}} d x=-\frac{1}{\sqrt{a^{2}+x^{2}}}+C
\end{aligned}
$$

TABLE 27-1 Summary of Right-hand Rules (= RHR)
$\left.\begin{array}{l|c|c|ll}\hline \text { Physical Situation } & \text { Example } & \text { How to Orient Right Hand } & \text { Result } \\ \hline \begin{array}{l}\text { 1. Magnetic field produced by } \\ \text { current } \\ \text { (RHR-1) }\end{array} & & \begin{array}{l}\text { Wrap fingers around wire } \\ \text { with thumb pointing in } \\ \text { direction of current } I\end{array} & \text { Fingers point in direction of } \overrightarrow{\mathbf{B}} \\ \hline \begin{array}{l}\text { 2. Force on electric current } I \\ \text { due to magnetic field } \\ \text { (RHR-2) }\end{array} & & \overrightarrow{\mathbf{F}} & & \text { Fig. 27-8c }\end{array}\right]$

