Problems	Points
Problem 1	/10
Problem 2	/10
Problem 3	/10
Problem 4	/10
Problem 5	/10
Problem 6	/10
Problem 7	/10
Problem 8	/10
Problem 9	/10
Problem 10	/10
Problem 11 [extra-	/10
credit]	
Total	/100

Take-home Exam Issued: Tuesday October 15 at 11:10 AM Due: Wednesday October 30 at 11:59 PM on bcourses (upload in PDF format)

# **Instructions:**

- You are free to consult your notes from the class, suggested readings, and other resources available on PubMed.
- Honor Code: I have not given or received aid in this examination. I have taken an active part in seeing to it that others as well as myself uphold the spirit and letter of this Honor Code.

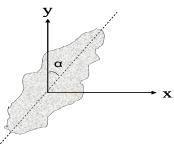
Name:

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**PROBLEM 1 (Relevance of** *Biomechanics* in Biomedical Engineering). Write a short (<2 pages) essay discussing the relevance of Biomechanics in bioengineering applications, ranging from "uncovering the causes of human diseases" to "therapeutical approaches" e.g. medical devices/implants, tissue engineering and regenerative medicine, etc. Feel free to include hand-drawn schematics.

**PROBLEM** 2 (Stress Transformation). You are studying a cell in a tissue – for simplicity's sake, the tissue is assumed to be 2D. For the given coordinate system, the state of stress in the tissue can be characterized by  $\sigma_{xx} = 170$  kPa,  $\sigma_{yy} = 30$  kPa, and  $\sigma_{xy} = 45$  kPa. The object in grey is a cell residing in the tissue, and the dotted line indicates the line that the major axis of the cell is oriented with – y simply, the way the cell is "pointing".  $\alpha = 30^{\circ}$ .

- a) Transform the coordinate system so that the new y (y') axis is oriented with the cell's major axis and calculate  $\sigma'_{xx}$ ,  $\sigma'_{yy}$ , and  $\sigma'_{xy}$ .
- b) Does the orientation of the cell line up with one of the principle stresses or the maximum shear? If not, which is it closest to?



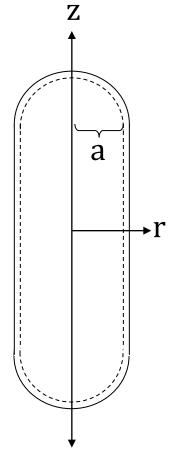
# **PROBLEM 3** (Compression Tests for Biomechanical Characterization of Soft Tissues and Biomaterials):

- a) Ghafar is a bioengineer working at a medical engineering company. He is tasked with characterizing a soft tissue specimen. Specifically, he aims to find the Young's modulus of elasticity for a tissue specimen. He sets out to do a *confined compression* test as follows. He cuts out a cylindrical piece of this tissue specimen and put it inside a metallic cylinder (with the same diameter as the soft tissue piece). The metallic ring is put around the tissue piece to hold it in place, while compression test is taking place. Then the two ends of the tissue are compressed and the strain is measured. Let's assume the loading direction is x. Can he find the Young's modulus by dividing the applied stress ( $\sigma_{xx}$ ) by the measured strain ( $\varepsilon_{xx}$ )? Why? If not, how can he find the actual Young's modulus? *Please answer thoroughly using the equations you have learned and clearly state your assumptions*.
- b) Minhee is a bioengineer working in an orthopedic research laboratory. She is tasked with characterizing a bone tissue. She designs an *unconfined compression* test as follows. She cuts out a cubic piece of the bone tissue with each side being of length L. Minhee has access to strain gauges and a device that can apply a compressive force F on the two sides of the cube.
  - i. Find stress in the loading direction.
  - ii. Describe the process of measuring strain
  - iii. Find Young's modulus of elasticity for the bone tissue in terms of calculated values of stress and strain.

Clearly state your assumptions.

**PROBLEM 4 (Pressure Vessel).** Consider a biomedical device which is long, cylindrical, pressurized, and *capped* at both ends by a hemisphere. When depressurized, the radius a is 3 cm and the thickness of the wall is 1 mm. When pressurized at 340 mmHg, the radius a is 4.2 cm and the thickness of the wall is 0.7 mm.

- a) Calculate  $\sigma_{\theta\theta}$  (in Pa) in the pressurized cylinder wall.
- b) Calculate  $\sigma_{zz}$  (in Pa) in the pressurized cylinder wall, and *briefly* justify your choice of the equation used to calculate it.



**PROBLEM 5 (Equilibrium).** *Derive* (yes, please drive) the partial differential equations for equilibrium in continuum mechanics:

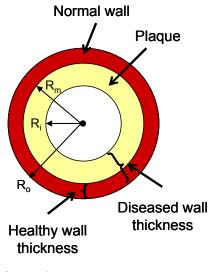
$$\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{yx}}{\partial y} + \frac{\partial \sigma_{zx}}{\partial z} + \rho g_x = 0$$
$$\frac{\partial \sigma_{xy}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \sigma_{zy}}{\partial z} + \rho g_y = 0$$
$$\frac{\partial \sigma_{xz}}{\partial x} + \frac{\partial \sigma_{yz}}{\partial y} + \frac{\partial \sigma_{zz}}{\partial z} + \rho g_z = 0$$

*Clearly state your assumption(s).* 

**PROBLEM 6 (Vascular Biomechanics).** This problem is an opportunity for you to develop biomechanical intuition and demonstrate critical thinking.

"It is known that plaque in a coronary artery can rupture due to high stress. Rupture of a plaque can lead to thrombosis and heart attack. What is the stress distribution in the human coronary artery pictured at right? By analyzing the stress state, predict whether this patient is at risk for plaque rupture."

What kinds of information (biological and mechanical) would you need to know before you could answer this question? Explain why this information is necessary. Specifically, try to describe how you could use this information to come to a conclusion regarding the plaque.





- Browse to <u>https://www.ahajournals.org/doi/abs/10.1161/01.cir.84.6.2294</u> and determine a suitable radius for the proximal left anterior descending artery. Choose whichever age group you would like.
- Browse to <a href="http://circ.ahajournals.org/cgi/reprint/102/5/506">http://circ.ahajournals.org/cgi/reprint/102/5/506</a> and use this article to determine a suitable wall thickness for a patient with coronary artery disease. We will assume that the thickness of normal tissue is the same as the wall thickness for a healthy patient, and use this to determine the thickness of the plaque (diseased normal thickness).

**PROBLEM 7** (Biomechanics for Design of Surgical Operations). J'Mya is cardiovascular surgeon conducting a coronary artery bypass graft surgery. During the operation, she is trying to cut a vein graft by inducing the least distortion (shear strain) in the vein graft but she doesn't care if the vein goes through a large contraction (normal strain) due to cutting. What is the best direction she can cut the vein for this purpose? Please fully explain by drawing the stresses that exist in a section of the vein and also making a complete analysis of the situation using the equations you have learned in the course.

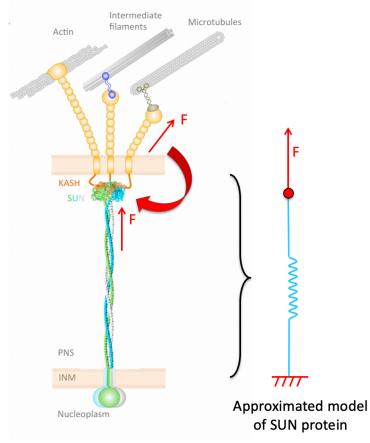
**PROBLEM 8 (Stress Shielding).** A total hip replacement is performed on a patient with exceptionally thin bones. The cross-sectional area of the bone is only 1/3rd of the cross-sectional area of the necessary implant. If the elastic modulus of the bone is 17 GPa, what would the elastic modulus of the implant material have to be to keep the stress shielding

 $f_p/f$  less than 70% (i.e.  $f_b/f$  above 30%)? Name a real material with (approximately) appropriate material properties (if such a thing exists).

**PROBLEM 9** (*Feel the Force*!). Read the *Current Opinion* minireview article titled "The Nucleus Feels the Force, LINCed In or Not!" by Jahed<sup>1</sup> & Mofrad. Briefly (in 1-2 pages) discuss various biomechanical ways the nucleus feels the force transmitted to it via the cell and surrounding tissue under strain or stress. Don't hesitate to include (hand-drawn) schematics.

<sup>&</sup>lt;sup>1</sup> Link to the minireview article: <u>https://www.sciencedirect.com/science/article/pii/S0955067418301558</u>

**PROBLEM 10 (Remember the** *Good Old Days).* Read the Jahed<sup>2</sup> & Mofrad's *Extreme Mechanics Letters* article titled "Mechanical LINCs of the nuclear envelope: Where SUN meets KASH". As discussed in the article, the linker of the nucleoskeleton and cytoskeleton (LINC complex) spans the nuclear envelope and is subjected to cytoskeletal forces. The diagram on the right shows the components of the LINC complex at the nuclear envelope. The long coiled-coil region of SUN protein behaves like and elastic rod and can be approximated by a simple linear spring as shown in the figure on the right. The spring-like behavior of the SUN protein allows changes in the distance between the inner and outer nuclear membranes (INM and ONM). Using the simple linear spring model on the right, determine the modulus of elasticity of SUN proteins if the distance between the INM and ONM can change from 30nm to 40nm in response to 0.05pN of cytoskeletal force. The cross-sectional diameter of the coiled coils of SUN is approximately 20Å.



<sup>&</sup>lt;sup>2</sup> Link to the review article: <u>https://www.sciencedirect.com/science/article/pii/S2352431617301876</u>

**PROBLEM 11 (Design a** *Class Competition Project* for Biomechanics) [Extra-credit Educational Contributions to Improving this Course]. Design a hands-on class competition for a Biomechanics class. Use your knowledge in the course topics, and of course your creativity and imagination, to devise the details of such a competition by providing (i) a detailed protocol for the competition, (ii) the materials to be used, and most importantly (iii) an "objective" mechanism to easily rank (referee) the work of the best group. Of course, this objective mechanism needs to be biomechanically relevant in its broadest sense of scales, from molecular and cell to tissue, and organ biomechanics.

[Hint: A similar concept exists in Structural Mechanics courses, where they compete for the best design and building of a bridge/structure, using spaghetti rods, elastic bands, glues, etc. There, the objective mechanism for selection of the winning model is simply the maximum load handled by the structures of a given set of materials. However, please don't let this example limit your imagination. Please think outside the box! Your proposed design for this competition will be graded based on its novelty, innovation, creativity, and of course its feasibility.]