Mathematics 54
Midterm 2, 6 November 2019
50 minutes, 50 points

NAME: ______________________ **ID:** ______________________

GSI: ____

INSTRUCTIONS:
You must justify your answers, except when told otherwise.
All the work for a question should be on the respective sheet.

This is a CLOSED BOOK examination, NO NOTES and NO CALCULATORS are allowed.
NO CELL PHONE or EARPHONE use is permitted.
Please turn in your finished examination to your GSI before leaving the room.

Q1	
Q2	
Q3	
Q4	
Tot	

Question 1. (15 points) Bubble in the correct answers, worth 1 point each. No justification necessary. Incorrect answers carry a 1-point penalty, so **random choices are not helpful**. You may leave any question blank for 0 points. You will not get a negative score on any group of five questions.

Ⓣ Ⓕ	An orthonormal collection of vectors in $\mathbb{R}^n$ is linearly independent.
Ⓣ Ⓕ	All eigenvalues of a symmetric matrix are real.
Ⓣ Ⓕ	Every linear system has a Least Squares solution.
Ⓣ Ⓕ	A square matrix is invertible precisely when 0 is an eigenvalue of it.
Ⓣ Ⓕ	If the columns of the real 2×2 matrix $M \neq I_2$ are orthonormal, then the matrix transformation $\mathbf{x} \mapsto M\mathbf{x}$ is a rotation or a reflection in $\mathbb{R}^2$.

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Ⓣ Ⓕ	The $n \times n$ matrix representing the orthogonal projection onto a line in $\mathbb{R}^n$ has rank one.
Ⓣ Ⓕ	The product of two orthogonal matrices of the same size is also an orthogonal matrix.
Ⓣ Ⓕ	The Least-Squares problem is finding a vector $\mathbf{x}$ which makes $A\mathbf{x}$ as close as possible to a given vector $\mathbf{b}$.
Ⓣ Ⓕ	If $(t-2)$ is a factor of the characteristic polynomial of A, then (-2) is an eigenvalue of A.
Ⓣ Ⓕ	The determinant of a square matrix is the product of its eigenvalues, included with their multiplicities.

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Ⓣ Ⓕ	If $\|\mathbf{u} - 2\mathbf{v}\| = \|\mathbf{u} + 2\mathbf{v}\|$, then the vectors $\mathbf{u}$ and $\mathbf{v}$ are orthogonal.
Ⓣ Ⓕ	If a vector $\mathbf{v}$ is orthogonal to every column of the matrix A, then $\mathbf{v}^T$ is in the left nullspace of A.
Ⓣ Ⓕ	Similar matrices have the same eigenvectors.
Ⓣ Ⓕ	If $AS = S$, then every nonzero column of S is an eigenvector of A.
Ⓣ Ⓕ	If $A^T A$ is the identity matrix, then the transformation $\mathbf{x} \mapsto A\mathbf{x}$ preserves lengths: $\|\mathbf{x}\| = \|A\mathbf{x}\|$ for all $\mathbf{x}$.

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Ⓣ Ⓕ	If a square matrix has orthonormal columns, then it also has orthonormal rows.
Ⓣ Ⓕ	If W is a subspace of $\mathbb{R}^n$ and $\mathbf{p}$ is in W and $\mathbf{q}$ in $W^\perp$, then $\|\mathbf{p} - \mathbf{q}\| = \|\mathbf{p}\|^2 + \|\mathbf{q}\|^2$.
Ⓣ Ⓕ	Every upper triangular matrix A is diagonalizable.
Ⓣ Ⓕ	A real 2×2 matrix which has one *non-real* eigenvalue must be diagonalizable over $\mathbb{C}$.
Ⓣ Ⓕ	If the columns of the $m \times n$ matrix A are linearly independent, then the matrix $A^T A$ is invertible.

Question 2. (11 points, 2+3+3+3)
Determine, for the matrix

$$A = \begin{bmatrix} 0.7 & 0.15 \\ 0.4 & 0.8 \end{bmatrix},$$

(a) the characteristic polynomial; (b) the eigenvalues; (c) an eigenbasis of $\mathbb{R}^2$; (d) a formula for A^n.
Note: You have opportunities to check your answers, and you should do so to avoid repeated penalties for mistakes carried forward. No square roots are needed.

Question 3. (12 points, 6+4+2)
For the vector $\mathbf{x} = [1, 2, 2, 3]^T$ and the subspace V of $\mathbb{R}^4$ defined by the equations

$$x_1 - x_2 + x_3 - x_4 = 0 \quad \text{and} \quad 3x_1 + x_2 - x_3 - 3x_4 = 0,$$

(a) Find the 4×4 matrix implementing the orthogonal projection of $\mathbb{R}^4$ onto V;
(b) Find the orthogonal projection of $\mathbf{x}$ onto V.
(c) Find the distance from $\mathbf{x}$ to V.
Explain your steps. You have opportunities to check your answer, do use them.
Hint: You may find it easier to project onto $V^\perp$.

Question 4. (12 points)
Set up and solve a consistent system of linear equations for the coefficients c_0, c_1 and c_2, whose solution gives the best fit to the relation $y = c_0 + c_1 x + c_2(x^2 - x)$, in the sense of least squares, with the following data points:

x	-1	0	1	2
y	1	0	1	2

THIS PAGE IS FOR ROUGH WORK (not graded)