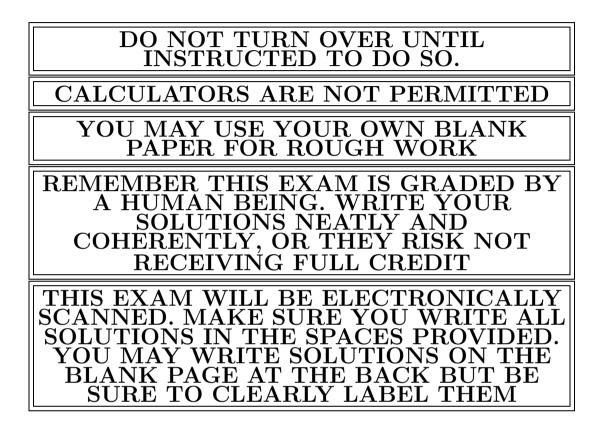
## MATH 54 FINAL EXAM (PRACTICE 1) PROFESSOR PAULIN



Name and section: \_\_\_\_\_

GSI's name: \_\_\_\_\_

This exam consists of 10 questions. Answer the questions in the spaces provided.

1. (25 points) (a) Let  $A = \begin{pmatrix} 1 & -1 & -1 & 0 & 2 & 1 \\ 0 & 0 & 1 & 0 & -1 & 1 \\ -1 & 1 & 0 & 0 & 0 & 1 \end{pmatrix}$ . Find a general solution to the homogeneous linear system with coefficient matrix A. Solution:

(b) What is  $Nullity(T_A)$ ? What is  $Rank(T_A)$ ? Solution:

2. (25 points) Let  $T: \mathbb{R}^3 \to \mathbb{R}^3$  be a linear transformation such that

$$T\begin{pmatrix}1\\2\\-1\end{pmatrix} = \begin{pmatrix}1\\2\\0\end{pmatrix}, T\begin{pmatrix}0\\-1\\1\end{pmatrix} = \begin{pmatrix}1\\1\\2\end{pmatrix}, T\begin{pmatrix}1\\1\\1\end{pmatrix} = \begin{pmatrix}-2\\0\\1\end{pmatrix}$$

Find the standard matrix of *T*. Is *T* one-to-one? Solution:

3. (25 points) Let A be a  $4 \times 3$  matrix and B be a  $3 \times 2$  matrix. Show that if the columns of A and B are linearly independent then the columns of AB are linearly independent. Hint: Consider the linear transformations associated to these matrices.

4. (25 points) Let V be a vector space with bases  $B = \{\underline{\mathbf{b}}_1, \underline{\mathbf{b}}_2, \underline{\mathbf{b}}_3\}$  and  $C = \{\underline{\mathbf{c}}_1, \underline{\mathbf{c}}_2, \underline{\mathbf{c}}_3\}$ , where

$$\underline{\mathbf{c}}_1 = \underline{\mathbf{b}}_1 - \underline{\mathbf{b}}_2, \quad \underline{\mathbf{c}}_2 = \underline{\mathbf{b}}_1 + \underline{\mathbf{b}}_3, \quad \underline{\mathbf{c}}_3 = \underline{\mathbf{b}}_1 + \underline{\mathbf{b}}_2 + \underline{\mathbf{b}}_3$$
  
If  $(\underline{\mathbf{x}})_B = \begin{pmatrix} 1\\1\\-1 \end{pmatrix}$ , what is  $(\underline{\mathbf{x}})_C$ ?

5. (25 points) Give an example of a non-diagonalizable matrix with only real eigenvalues. Carefully justify your answer.

6. (25 points) Compute the minimum distance between  $\begin{pmatrix} 1\\0\\-1\\0 \end{pmatrix}$  and

$$Nul \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 6 & 9 & 12 \\ 2 & 4 & 6 & 8 \end{pmatrix}$$

7. (25 points) Perform a singular-value decomposition of the matrix  $% \left( \frac{1}{2} \right) = 0$ 

$$\begin{pmatrix} 1 & 2 \\ 1 & 2 \\ 1 & 2 \end{pmatrix}.$$

Solution (continued) :

8. (25 points) Find a general solution to the following differential equation

$$y'' - 2y' + y = 12t^2e^t$$

Solution (continued) :

9. (25 points) Consider the following  $\mathbb{R}^3$ -valued function on  $\mathbb{R}$ :

$$\begin{pmatrix} t \\ t \\ t^2 \end{pmatrix}, \begin{pmatrix} t^2 \\ t^2 \\ t^3 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ t \end{pmatrix}.$$

Determine if these vector-valued functions are linearly independent. Are they solutions to some  $3 \times 3$  homogeneous linear system of differential equations? Carefully justify your answers.

10. (25 points) Calculate the sine Fourier series of the function f(x) = x, on the interval  $[0, \pi]$ . Use this to prove that

$$\frac{1}{1} - \frac{1}{3} + \frac{1}{5} + \frac{1}{7} - \frac{1}{9} + \dots = \frac{\pi}{4}$$