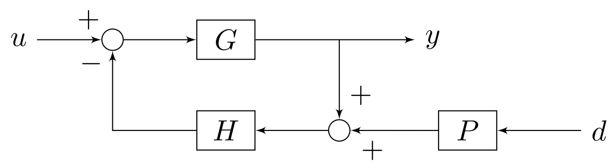
Problem # 1 (7 points)

(a) actuator, (b) disturbance, (c) noise, (d) controller, (e) reference, (f) sensor (g) plant

Problem # 2 *Short Questions* (4+4+4+4 = 16 points)



(a) Find the transfer function from d to y in the diagram above in terms of G, H and P.

We are asked to find the transfer function from d to y, so we can set $u = 0$. Observe that

$$y = -GH(y + Pd) \quad \Longrightarrow \quad y = \left[\frac{-GHP}{1+GH}\right] d$$

(b) Fill in the blanks:

Very accurate models are useful for simulation.
Very accurate models are bad for controller design.
Frequency response makes sense only for systems that are stable and ;uelinear and time - invariant.

(c) For the differential equation

$$\ddot{y} + 6\dot{y} + 100y = 33u$$

find the damping, natural frequency, pole locations and DC gain.

natural frequency = 10 rad/sec
damping = 0.3
poles are at $\xi\omega_n \pm \omega_n\sqrt{1-\xi^2} = -3 \pm j9$
DC gain = 0.33

(d) What is the solution of the ordinary differential equation

$$\frac{dy}{dt} = 1 \qquad \text{subject to the initial condition:} \quad y(0) = 5$$

$y(t) = t + 5$

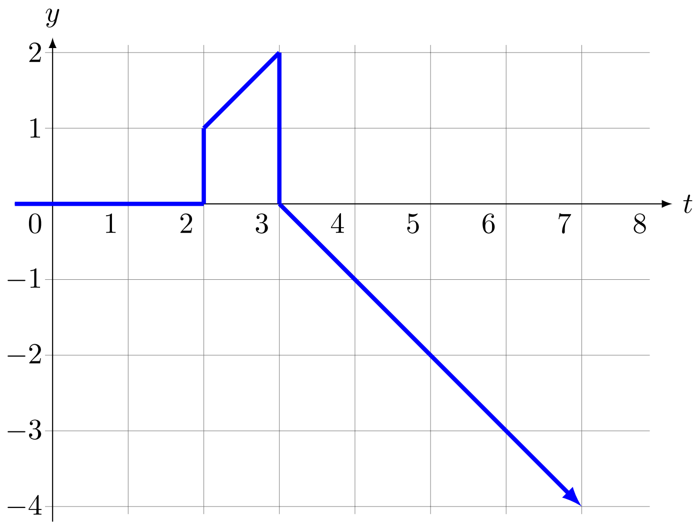
Problem # 3 (10 points)

Sketch the response of the system with transfer function $\left[\frac{s+1}{s}\right]$ to the input u shown below. Assume that the initial conditions are zero.

Notice that

$$y = \left[1 + \frac{1}{s}\right] u = u + \int u$$

The solution is plotted below.



Problem # 4 (12 points)

Below are 6 different input-output differential equation models.

A. $\ddot{y} + 0.4\dot{y} + y = 4\dot{u} - u$

B. $\ddot{y} + 8.4\dot{y} + 36y = -36u$

C. $\ddot{y} + 1.4\dot{y} + y = -5\dot{u} - u$

D. $\ddot{y} + 0.4\dot{y} + y = -4\dot{u}$

E. $\ddot{y} + 1.4\dot{y} + y = u$

F. $\ddot{y} + 2\dot{y} + 25y = 6\dot{u}$

The unit-step response with zero initial conditions for these models are shown below. Match these step responses with the models **A** through **F**.

No reasons are needed, but incorrect answers receive -2 points.

We first notice the following.

A. DC gain = -1, RHP zero so we must have undershoot.

B. DC gain = -1, $\xi\omega_n = 4.2$, so settling time $= 3/4.2 \approx 0.75$ sec

C. DC gain = -1, $\xi\omega_n = 0.7$, so settling time $= 3/0.7 \approx 4$ sec

D. DC gain = 0, $\xi\omega_n = 0.2$, so settling time $= 3/0.2 \approx 15$ sec

E. DC gain = 1

F. DC gain = 0, $\xi\omega_n = 1$, so settling time $= 3/1 \approx 3$ sec

Using this information, we can match the plots:
first column (top to bottom) D, B, E
second column (top to bottom) C, F, A

Problem # 5 (6+6 = 12 points)

Consider the first order LTI system $H(s) = \dfrac{b}{(s+a)}$. Suppose $a > 0$ and $b > 0$.
We observe two facts:

- in steady-state, the input $u(t) = \sin(t)$ is amplified by a factor of $\sqrt{2}$
- in steady-state, the input $u(t) = \sin(2t)$ is amplified by a factor of 1

Find a and b.
You must provide explanations or show your work for partial credit.

From the first fact, we conclude that

$$|H(j)| = \left|\frac{b}{j+a}\right| = \sqrt{2} \quad \Longrightarrow \quad b^2 = 2(1+a^2)$$

From the second fact, we conclude that

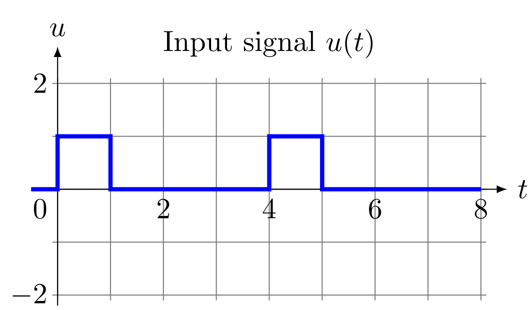
$$|H(2j)| = \left|\frac{b}{2j+a}\right| = 1 \quad \Longrightarrow \quad b^2 = 4 + a^2$$

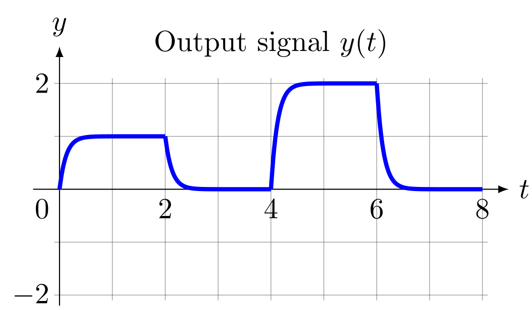
We have two equations in a, b and we can solve to get

$$a = \sqrt{2},\ b = \sqrt{6}$$

Problem # 6 (5 + 5 = 10 points)

(a) Consider some stable system H.
When we apply the input u to this system, we observe the output y.
The input u and output y are plotted below.
Assume that the transients settle quickly and H has no delays.
Explain why this system is not linear and time-invariant (LTI).

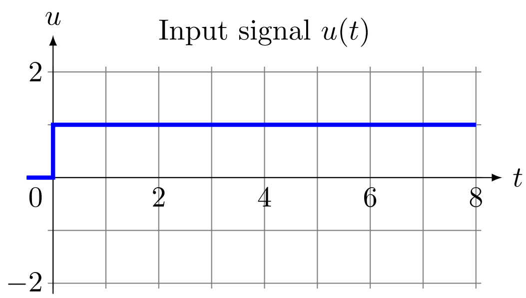


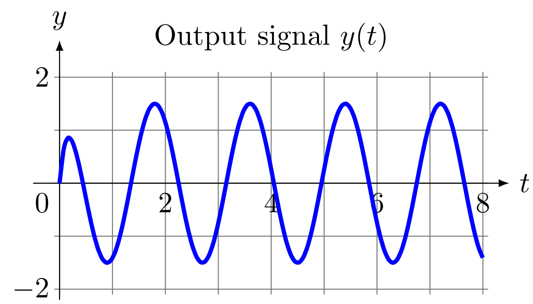


There are 2 pulses in the input, and 2 pulses in the output. Since transients decay quickly, we can say that the first input pulse causes the first output pulse, while the second input pulse causes the second output pulse. Notice that the second input pulse is a delayed version of the first one, but the corresponding output pulse is not a delayed version of the first one. So the system is not time invariant.

Also, the second output pulse is twice the height of teh first one. So the system is not linear.

(b) Consider some stable system H.
When we apply the input u to this system, we observe the output y.
The input u and output y are plotted below.
Assume that the transients settle quickly and H has no delays.
Explain why this system is not linear and time-invariant (LTI).





For LTI systems, if we inject a sine wave, we will get out a sine wave of the same frequency possibly amplified or attenuated. In this example, we are injecting a step input (sine wave of frequency 0) and getting a sine wave out of a different (nonzero) frequency. System cannot be LTI.

Problem # 7 (12 points)

Four companies have designed a cruise controller for a car.
The resulting closed loop transfer function matrices are:

$$\text{Design A} \qquad y = \left[\frac{3}{s+3}\right] r + \left[\frac{1}{s+10}\right] d$$

$$\text{Design B} \qquad y = \left[\frac{3}{s+3}\right] r + \left[\frac{s}{s+10}\right] d$$

$$\text{Design C} \qquad y = \left[\frac{3(s-1)}{s-3}\right] r + \left[\frac{s}{s+10}\right] d$$

$$\text{Design D} \qquad y = \left[\frac{70}{(s+3)(s^2+10s+25)}\right] r + \left[\frac{s}{s+10}\right] d$$

Here, r is the reference, d is the disturbance, and y is the car speed.
Which design would you choose? Explain your answers in the boxes below.

Design A: DC gain from r to y =1 which gives perfect tracking. DC gain from d to $y = 0.1$, so we do not reject constant disturbances perfectly.
Design B: DC gain from r to y =1 which gives perfect tracking. DC gain from d to $y = 0$, so we reject constant disturbances perfectly. This is the best.
Design C: Unstable. SO it can be eliminated immediately!
Design D: DC gain from r to $y70/75 \neq 1$ which does not gives perfect tracking. DC gain from d to $y = 0$, so we reject constant disturbances perfectly.