## NAME:

ID \# :

| $\# 1$ | $\# 2$ | $\# 3$ | $\# 4$ | $\# 5$ | $\# 6$ | $\# 7$ | TOTAL |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  |  |
| 7 | 16 | 10 | 12 | 12 | 10 | 12 | 79 |

## Instructions:

1 Write your name and student ID number.
2 Read the questions carefully.
3 Write your solution clearly. Please, please please ...
4 This exam has 7 questions worth 79 points, so you should proceed at approximately 1 point per minute.

Problem \# 1 (7 points)

Mix and match the following.
Enter your answers (i.e. the letters A through G) in the boxes provided.
Incorrect answers get -1 points.

sensor

actuator
$\square$ disturbance
$\square$ controller

noise
$\square$
A. a device that can alter the behavior of the plant, typically by electro-mechanical means
B. unpredictable phenomena from the outside world that affect the plant
C. errors in measurement, errors due to quantization
D. the device that processes measurements and commands and adjusts the actuators
E. a signal from the outside world that we would like the controlled variables to look like
F. device to measure physical quantities
G. physical system to be controlled

Problem \# 2 Short Questions ( $4+4+4+4=16$ points)

(a) Find the transfer function from $d$ to $y$ in the diagram above in terms of $G, H$ and $P$.

Answer:
(b) Fill in the blanks:

Very accurate models are useful for $\square$
Very accurate models are bad for $\square$
Frequency response makes sense only for systems that are:

(c) For the differential equation

$$
\ddot{y}+6 \dot{y}+100 y=33 u
$$

find the damping, natural frequency, pole locations and DC gain.

| damping: | natural freq: | poles: |
| :--- | :--- | :--- |

(d) What is the solution of the ordinary differential equation

$$
\frac{d y}{d t}=1 \quad \text { subject to the initial condition: } y(0)=5
$$

$$
y(t)=
$$

Problem \# 3 (10 points)
Sketch the response of the system with transfer function $\left[\frac{s+1}{s}\right]$ to the input $u$ shown below. Assume that the initial conditions are zero.


## Draw your answer here:



Problem \# 4 (12 points)
Below are 6 different input-output differential equation models.
A. $\ddot{y}+0.4 \dot{y}+y=4 \dot{u}-u$
B. $\ddot{y}+8.4 \dot{y}+36 y=-36 u$
C. $\ddot{y}+1.4 \dot{y}+y=-5 \dot{u}-u$
D. $\ddot{y}+0.4 \dot{y}+y=-4 \dot{u}$
E. $\ddot{y}+1.4 \dot{y}+y=u$
F. $\ddot{y}+2 \dot{y}+25 y=6 \dot{u}$

The unit-step response with zero initial conditions for these models are shown below. Match these step responses with the models $\mathbf{A}$ through $\mathbf{F}$.
No reasons are needed, but incorrect answers receive - 2 points.






$\square$
$\square$

Problem \# 5 (6+6 = 12 points)
Consider the first order LTI system $H(s)=\frac{b}{(s+a)}$. Suppose $a>0$ and $b>0$.
We observe two facts:

- in steady-state, the input $u(t)=\sin (t)$ is amplified by a factor of $\sqrt{2}$
- in steady-state, the input $u(t)=\sin (2 t)$ is amplified by a factor of 1

Find $a$ and $b$.
You must provide explanations or show your work for partial credit.
$a=$
$b=$

Problem \#6 ( $5+5=10$ points)
(a) Consider some stable system $H$.

When we apply the input $u$ to this system, we observe the output $y$.
The input $u$ and output $y$ are plotted below.
Assume that the transients settle quickly and $H$ has no delays.
Explain why this system is not linear and time-invariant (LTI).



Answer (2 sentences max):
(b) Consider some stable system $H$.

When we apply the input $u$ to this system, we observe the output $y$.
The input $u$ and output $y$ are plotted below.
Assume that the transients settle quickly and $H$ has no delays.
Explain why this system is not linear and time-invariant (LTI).



Answer (2 sentences max):

## Problem \# 7 (12 points)

Four companies have designed a cruise controller for a car.
The resulting closed loop transfer function matrices are:
Design A $\quad y=\left[\frac{3}{s+3}\right] r+\left[\frac{1}{s+10}\right] d$
Design B $\quad y=\left[\frac{3}{s+3}\right] r+\left[\frac{s}{s+10}\right] d$
Design C $\quad y=\left[\frac{3(s-1)}{s-3}\right] r+\left[\frac{s}{s+10}\right] d$
Design D $\quad y=\left[\frac{70}{(s+3)\left(s^{2}+10 s+25\right)}\right] r+\left[\frac{s}{s+10}\right] d$
Here, $r$ is the reference, $d$ is the disturbance, and $y$ is the car speed.
Which design would you choose? Explain your answers in the boxes below.

| Design A: Accept or Reject (circle one) <br> Reason one (1 sentence max): Reason two (may not be needed, 1 sentence max): |
| :--- |
| Reason one (1 sentence max): Reason two (may not be needed, 1 sentence max): <br> Design C: Accept or Reject (circle one)  <br> Reason one (1 sentence max): Reason two (may not be needed, 1 sentence max): <br> Resign D: Accept or Reject (circle one)  <br> Reason one (1 sentence max): Reason two (may not be needed, 1 sentence max): |

