PHYSICS 7A, Lecture 1 – Spring 2019
Midterm 1, C. Bordel
Monday, Feb. 25th, 7-9 pm

- Student name:
- Student ID #:
- Discussion section #:
- Name of your GSI:
- Day/time of your DS:

Physics Instructions

All moving objects can be considered as point masses. You may assume that air resistance is negligible (unless specified otherwise) and that the acceleration due to gravity has constant magnitude g.
Remember that you need to show your work and justify your answers in order to get full credit!

Math Information Sheet

- Solutions to equation $ax^2 + bx + c = 0$ are: $x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$
- $\sin 90° = \cos 0° = -\cos 180° = 1$
- $\sin 0° = \cos 90° = \sin 180° = 0$
- $\sin 45° = \cos 45° = \sqrt{2}/2$
- $\cos(180° - \theta) = -\cos\theta$
- $\sin(180° - \theta) = \sin\theta$
- $\cos(90° + \theta) = -\cos(90° - \theta) = -\sin\theta$
- $\sin(90° + \theta) = \sin(90° - \theta) = \cos\theta$
- $\cos^2\theta + \sin^2\theta = 1$
- Circumference of circle: $2\pi R$
- Area of disk: πR^2
- Surface area of sphere: $4\pi R^2$
- Volume of sphere: $4\pi R^3/3$
- Volume of cylinder: $\pi R^2 h$
- Lateral area of cylinder: $2\pi Rh$

Name:

Problem 1 – Miniature rocket launch (25 pts)

Children are playing with a miniature rocket which is initially at rest on the ground. At time $t=0$, the propeller is turned on and the toy is launched vertically upward (Fig.1), experiencing a constant acceleration of magnitude a with respect to the ground. At time t_1 after the launch, the propeller is remotely turned off. Let upward be the positive orientation of the y-axis, attached to the ground.

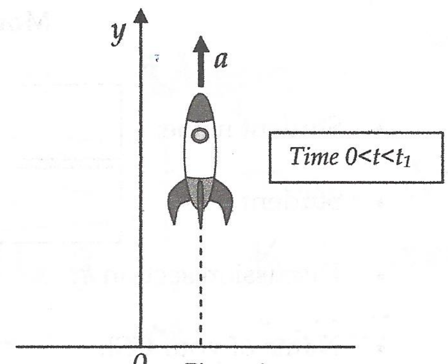


Figure 1

a- What is the instantaneous velocity v_1 of the rocket, with respect to the ground, when the propeller is turned off?

From time $t=0$ to t_1, the rocket has constant acceleration a, starting from rest on the ground.

Then $v(t) = at + \cancel{v_0}^{\,0}$ and $\boxed{v_1 = v(t_1) = at_1}$

b- What maximum height h above the ground does the rocket reach?

- During the accelerated phase, the rocket travels distance $y_1 = y(t_1) = \frac{1}{2}at_1^2 + \underbrace{\cancel{v_0}^{\,0} t_1 + \cancel{y_0}^{\,0}}_{\text{starts from rest on the ground}}$
- After propeller is turned off: motion with constant acceleration $(-g)$.

Max. height corresponds to $v=0$ (no horizontal motion) so we can use kinematic eq. for const. acceleration

$$\frac{\cancel{v_{top}^2}^{\,0} - v_1^2}{2(-g)} = y_{top} - y_1 \text{ then } y_{top} = \frac{1}{2}at_1^2 + \frac{a^2t_1^2}{2g}$$

$$\boxed{h = y_{top} = \frac{1}{2}at_1^2\left(1+\frac{a}{g}\right)}$$

Name:

c- How long does it take the rocket to reach the highest point from the moment it takes off?

Total time $= t_1 + \underbrace{t_{top} - t_1}_{t_2 = \text{time in free fall}} = t_{max}$

After propeller is turned off: $v(t) = v_1 - g t$ ← const.

$v_{top} = v(t_2) = 0$ then $t_2 = \frac{v_1}{g}$

therefore $\boxed{t_{max} = t_1 + \frac{a t_1}{g} = \left(1 + \frac{a}{g}\right) t_1}$

d- What is the velocity v_f of the rocket, with respect to the ground, just before it hits the ground?

From max. height h to the ground, displacement is $-h$, initial velocity $v_i = 0$, final velocity v_f and acceleration $-g$ (constant).

Then we can use: $\Delta y = \frac{v_f^2 - v_i^2}{2a}$

$-h = \frac{v_f^2 - 0}{-2g} \Leftrightarrow v_f^2 = 2gh$

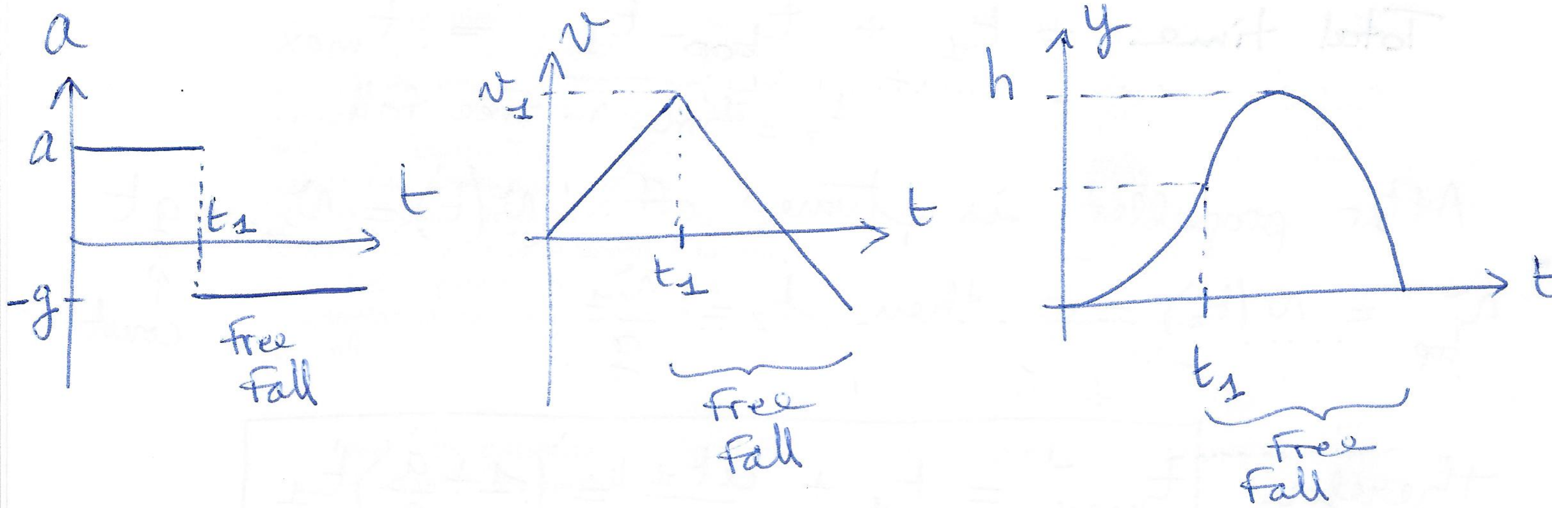
$\Leftrightarrow v_f = \pm \sqrt{2gh}$

We choose the negative solution since object is moving along $(-y)$.

$\boxed{v_f = -t_1 \sqrt{a(a+g)}}$

Name:

e- Make three qualitative plots representing the acceleration, velocity and position of the rocket as a function of time, using the coordinate system defined in Fig.1.



Problem 2 – Dropping a package (25 pts)

Called on a rescue mission, a helicopter is in vertical descending motion with constant vertical speed v_h right above a boat drifting to the right at speed v_d (Fig.2). Both speeds are measured with respect to the ground. At time $t=0$, the helicopter is at an altitude H above the boat and a package of emergency supplies is thrown horizontally with respect to the helicopter.

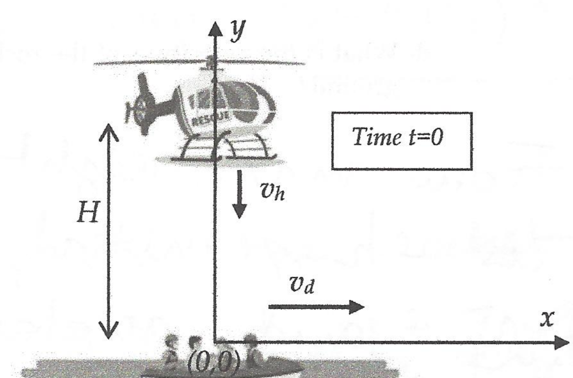


Figure 2

a- What should be the initial speed v_p and direction (left, right) of the package with respect to the helicopter to ensure that it lands on the boat?

Package would land on the boat if it has the same horizontal velocity as the boat (with respect to the shore).

So the initial velocity of the package with respect to the helicopter (static in x direction) should be: $\boxed{\vec{v}_p = v_d\,\hat{i}}$ pointing to the right

Name:

b- Assuming that the previous condition is met, how long does it take for the package to reach the boat?

Package = projectile in free fall with const. acceleration $(-g)$.

We can use kinematic eq: $y(t) = -\frac{1}{2}gt^2 + v_y t + H$

initial vertical velocity same as helicopter: $-v_h$

then $y(t) = -\frac{1}{2}gt^2 - v_h t + H$ and $y_f = 0$

solutions are: $t_f = \dfrac{v_h \pm \sqrt{v_h^2 + 2gH}}{-g}$

Time must be positive so we choose: $\boxed{t_f = \dfrac{-v_h + \sqrt{v_h^2 + 2gH}}{g}}$

c- Establish the equation $y=f(x)$ describing the trajectory of the package for an observer watching the maneuver from the shore.

Equations of the motion are
$$\begin{cases} (1)\ x(t) = v_d t \\ (2)\ y(t) = -\frac{1}{2}gt^2 - v_h t + H \end{cases}$$

Eq. (1) gives: $t = \dfrac{x}{v_d}$

Substitution into (2) gives:

$$\boxed{y(x) = -\frac{1}{2}g\left(\frac{x}{v_d}\right)^2 - v_h \frac{x}{v_d} + H}$$

Parabola

downward concavity

negative initial slope indicates downward initial velocity

Name:

d- What is the total horizontal distance traveled by the package for the previous observer?

Total horizontal distance is obtained by $x(t_f) = v_d t_f$

then $\boxed{x_{max} = v_d \left(\frac{-v_h + \sqrt{v_h^2 + 2gH}}{g} \right)}$

e- Find an expression for the angle between the velocity and the horizontal direction when the package lands on the boat. Specify whether the velocity points above or below the horizontal.

Angle θ between $\vec{v}$ and horizontal verifies:

$$\tan\theta = \frac{v_y}{v_x}$$

so $\theta_f = \tan^{-1}\left(\frac{v_{y_f}}{v_{x_f}}\right) = \tan^{-1}\left(\frac{-v_h - g\frac{-v_h + \sqrt{v_h^2 + 2gH}}{g}}{v_d}\right)$

$$\boxed{\theta_f = \tan^{-1}\left(\frac{-\sqrt{v_h^2 + 2gH}}{v_d}\right)}$$

$\vec{v}_f$ points below the horizontal axis

Name:

Problem 3 – Block and bucket (25 pts)

A block of mass m lies on a rough horizontal table. It is connected to a bucket of mass M by an ideal rope going over a massless and frictionless pulley (Fig.3). The coefficients of static and kinetic friction between the block and the table are μ_s and μ_k respectively. The bucket, initially empty, remains at rest when the block is released.

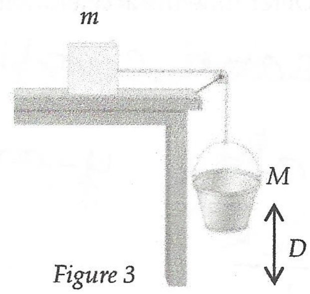
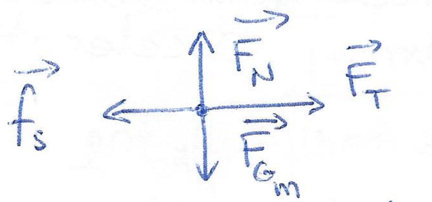
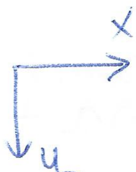


Figure 3

a- Define the coordinate system(s) of your choice and determine the tension acting at each end of the rope.



FBD of block



FBD of bucket

Newton's 2nd law for const. mass and no acceleration gives $\vec{F}_{net} = \vec{0}$ for each object.

Bucket $/\!/ y$: $Mg - F_T = 0$ and rope is ideal so F_T const. throughout therefore $\boxed{F_T = Mg}$

b- Determine the minimum mass M_{min} of the bucket that allows the system to be in motion.

Min. mass that allows block to move is also max. mass that allows block to remain stationary.

Latter statement gives: $F_T = f_s$ based on NSL for the block along x.

We know that $f_{s_{max}} = \mu_s F_N$ and NSL $/\!/ y$ gives $F_N = mg$ therefore $M\cancel{g} = \mu_s m\cancel{g}$.

Min. mass allowing motion is: $\boxed{M_{min} = \mu_s m}$

Name:

The bucket is now filled with water so that its mass M is larger than M_{min}.

c- Determine the acceleration of the block and that of the bucket.

Using same coord. syst. as previously, we project NSL along y-axis for bucket and along x-axis for block.

(1) $F_T - f_k = m\, a_{block}$

(2) $Mg - F_T = M\, a_{bucket}$

Ideal rope gives same F_T and same acceleration

Eq. (1) gives $F_T = ma + \mu_k F_N = ma + \mu_k mg$

substitution in (2) gives: $Mg - ma - \mu_k mg = Ma$

then $\boxed{a = \dfrac{(M - \mu_k m)g}{M+m}} > 0$

d- Determine the tension acting at each end of the rope.

Back to Eq. (2) with known a gives:

$$F_T = Mg - Ma$$

$$= Mg - \frac{M}{M+m}(M - \mu_k m)g$$

$$\boxed{F_T = \frac{Mm(1+\mu_k)g}{M+m}}$$

Name:

e- Assuming that the block is released from rest when the bucket is initially a distance D above the floor, how much time does it take the bucket to reach the floor?

a = const. so we can use kinematic equation

$$\Delta y = \frac{1}{2} a t^2 + v_0 t = D$$

then $t = +\sqrt{\frac{2D}{a}}$

$$\boxed{t = \sqrt{\frac{2D\,(M+m)}{(M-\mu_k m)\,g}}}$$

Time to reach the floor

<u>Problem 4</u> – Spinning block (25 pts)

A block of mass M is attached to an ideal spring of stiffness constant k, whose other end is attached to a vertical post. The whole spring-mass system rests on a horizontal frictionless table and spins at constant angular speed ω (Fig.4.1). You may assume that the spring does not bend.

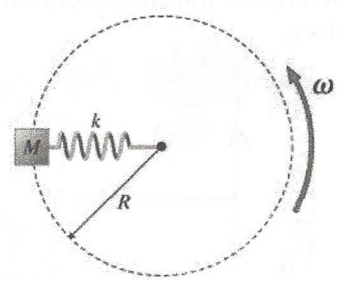
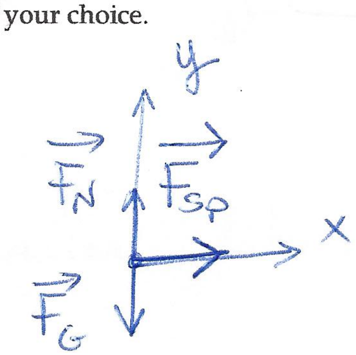


Figure 4.1 (top view)

a- Draw the free body diagram of the block (side view) and define the coordinate system of your choice.



We choose x-axis pointing radially inward and y-axis upward.

Name:

b- Determine the change in length ΔL experienced by the spring between its equilibrium length (when it is neither compressed nor stretched) and the length during its horizontal rotation at constant speed. → Uniform Circular Motion

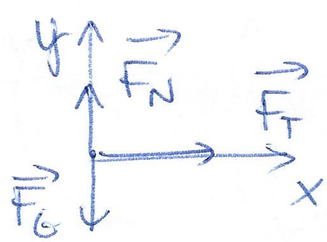
$\vec{F}_{net} = M\vec{a}$ projected along x-axis

gives : $-k\Delta x = M\frac{v^2}{R}$ where $\frac{v^2}{R}$ is the magnitude of the radial acceleration pointing inward. Note that Δx must be negative meaning spring stretches. Then change in length ΔL verifies $\Delta L = -\Delta x = \frac{Mv^2}{kR}$

We know that $v = R\omega$ so $\boxed{\Delta L = \frac{MR\omega^2}{k}}$

The spring is replaced by an ideal rope.

c- Determine the magnitude of the normal force and tension force experienced by the block.



Now the centripetal force is $\vec{F_T}$, and the block is still in uniform circular motion.

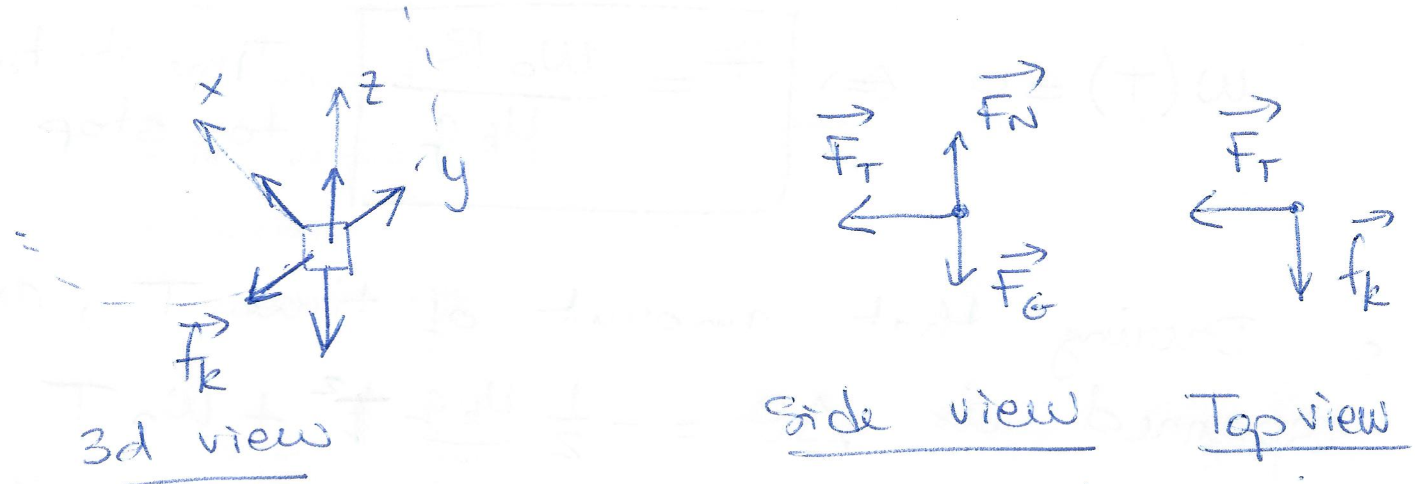
NSL // x : $\boxed{F_T = MR\omega^2}$

NSL // y : $\boxed{F_N = Mg}$

Name:

Now the block, initially launched with an angular speed ω_0, experiences kinetic friction during its circular motion.

d- Determine how the angular speed and tension vary as a function of time.



Choose x-axis along tension (inward), y-axis in the direction of motion and z-axis upward.

NSL gives : $/\!/x:\ F_T(t) = MR\omega^2_{(t)}$

$/\!/y:\ -\mu_k F_N = M\dfrac{dv}{dt} = MR\dfrac{d\omega}{dt}$

$/\!/z:\ \boxed{F_N = Mg}$

then $/\!/y$ gives $\dfrac{d\omega}{dt} = \dfrac{-\mu_k Mg}{MR} \Rightarrow \boxed{\omega(t) = -\dfrac{\mu_k g t}{R} + \omega_0}$

const.

Tension varies like : $\boxed{F_T(t) = MR\left(\omega_0 - \dfrac{\mu_k g t}{R}\right)^2}$

Both ω and F_T decrease as a function of time.

Name:

e- Determine how long it takes the block to get to a stop and what arc length it travels until then.

- $v = \omega = 0$ when block stops

$$\omega(T) = 0 \iff \boxed{T = \frac{\omega_o R}{\mu_k g}}$$ Time it takes to stop

- During that amount of time T, angle spanned is $\Delta\theta = -\frac{1}{2}\frac{\mu_k g}{R}T^2 + \omega_o T$

therefore distance traveled along circular path is:

$$\Delta s = R\,\Delta\theta = -\frac{1}{2}\mu_k g \frac{R^2\omega_o^2}{\mu_k^2 g^2} + \omega_o \frac{R\omega_o}{\mu_k g}$$

$$\boxed{\Delta s = \frac{R^2\omega_o^2}{2\mu_k g}}$$