ME 132, Fall 2015, Quiz # 1

# 1	# 2	# 3	# 4	#5	NAME
24	16	12	30	18	

Decent plot question from Spring 2012 Final. A couple of boring, but OK problems from Spring 14 final.

Rules:

1. 1 sheet of notes allowed, 8.5×11 inches. Both sides can be used.
2. Calculator is allowed. **Keep it in plain view on the desk next to you.**
3. **No laptops, phones, headphones, pads, tablets, or any other such device may be out. If such a device is seen after 1:10PM, your test will be confiscated, and you will get a 0 for the exam.**
4. **Sit with at least one open space between every student**
5. **The exam ends promptly at 2:00 PM**.
6. **Stop working, and turn in exams when notified.**

For complex numbers N and D

$$|ND| = |N| \cdot |D|, \quad \angle(ND) = \angle N + \angle D$$

Furthermore, if $D \neq 0$,

$$\angle \frac{1}{D} = -\angle D.$$

If D is real, and $D > 0$, then $\angle D = 0$.

1. Consider the delay-differential equation

$$\dot{x}(t) = A_1 x(t) + A_2 x(t - T)$$

where A_1, A_2 and T are real-valued constants. $T \geq 0$ is called the "delay." Depending on the values, there are 3 cases:

- The system is unstable for $T = 0$; or
- The system is stable for all $T \geq 0$; or
- The system is stable for $T = 0$, but unstable for some positive value of T. In this case, we are interested in the smallest $T > 0$ for which instability occurs, and the frequency of the nondecaying oscillation that occurs at this critical value of delay.

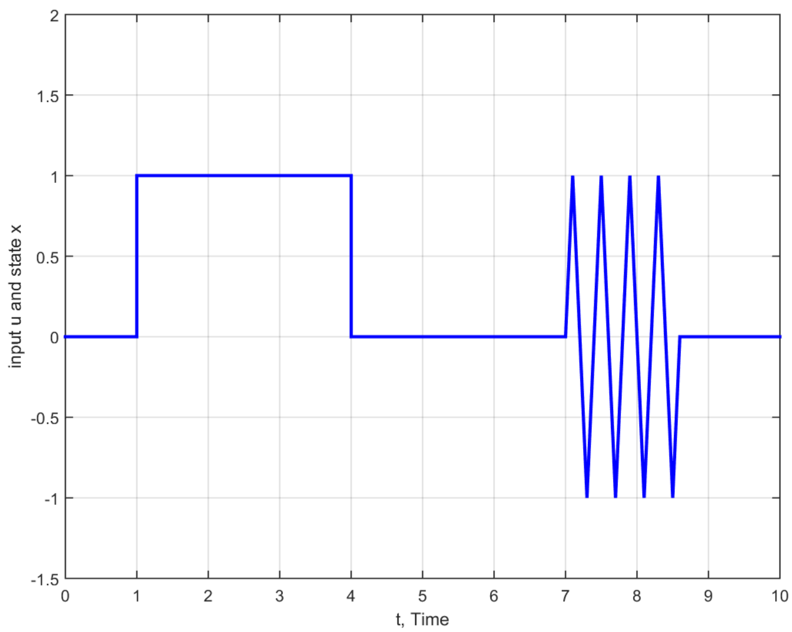
Fill in the table below. In each row, please mark/check one of the first three columns (from the three cases above). **If you check the 3rd column, then include numerical values in the 4th and 5th columns associated with the instability. Show work below.**

	unstable at $T = 0$	stable for all $T \geq 0$	stable at $T = 0$, but unstable at some finite $T > 0$	frequency at which instability occurs	smallest T at which instability occurs
$A_1 = -2$, $A_2 = 3$					
$A_1 = -4$, $A_2 = 1$					
$A_1 = -2$, $A_2 = -1$					
$A_1 = -1$, $A_2 = -2$					
$A_1 = 1$, $A_2 = -4$					
$A_1 = 0$, $A_2 = -1$					

2. A stable first-order system has

- a time-constant of 1;
- a steady-state gain of 1.5; and
- an instantaneous gain of -0.5.

(a) Sketch the approximate response of the system (starting from 0-initial condition) to the input shown. Remember that the steady-state gain is not equal to 1.



(b) Find values a, b, c, d such that system

$$\begin{aligned}\dot{x}(t) &= ax(t) + bu(t) \\ y(t) &= cx(t) + du(t)\end{aligned}$$

has the specified properties (ie., time-constant, steady-state gain, and instantaneous gain) as given above. **Hint:** Correct answer is not unique - different combinations of (a, b, c, d) all combine to achieve these 3 specified properties.

3. (a) Define the complex number $g = \frac{5}{j4+3}$.

   i. Find the value of $\mathbf{Re}(g)$

   ii. Find the value of $\mathbf{Im}(g)$

   iii. Find the value of $|g|$

   iv. Find the value of $\angle g$

(b) Sketch the final output (in the axes) of the Matlab code

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uH = @(z) sin(4*z);
fH = @(t,x) -3*x + 5*uH(t);
[tSol, xSol] = ode45(fH,[0 100],-6);
plot(tSol, uH(tSol),’--’, tSol, xSol);  % input=dashed; solution=solid
xlim((2*pi/4)*[20 21]);   % resets horizontal limits
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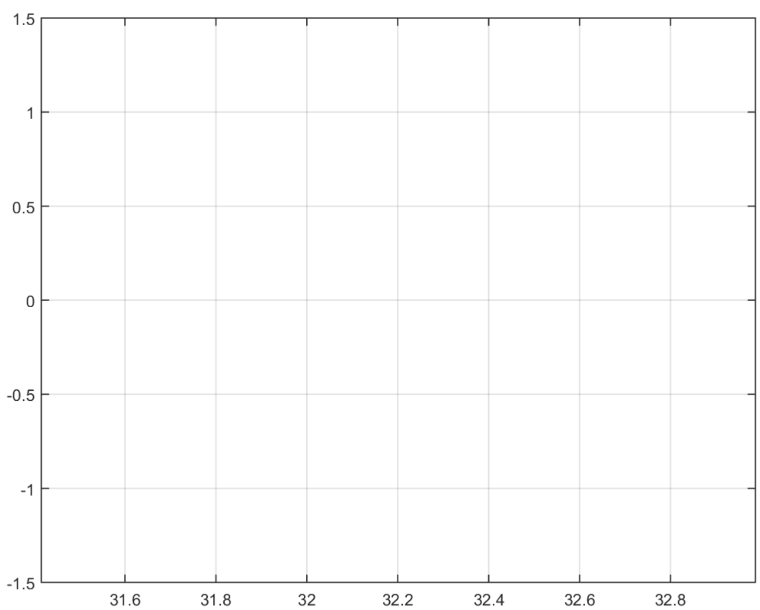


4. A process, with input u, disturbance d and output y is governed by

$$\dot{x}(t) = 2x(t) + d(t) + 3u(t), \qquad y(t) = x(t)$$

(a) Is the process stable?

(b) Suppose $x(0) = 1$, and $u(t) = d(t) \equiv 0$ for all $t \geq 0$. What is the solution $x(t)$ for $t \geq 0$.

(c) Consider a proportional-control strategy, $u(t) = K_1 r(t) + K_2 [r(t) - y(t)]$. Determine the closed-loop differential equation relating the variables (x, r, d).

(d) For what values of K_1 and K_2 is the closed-loop system stable?

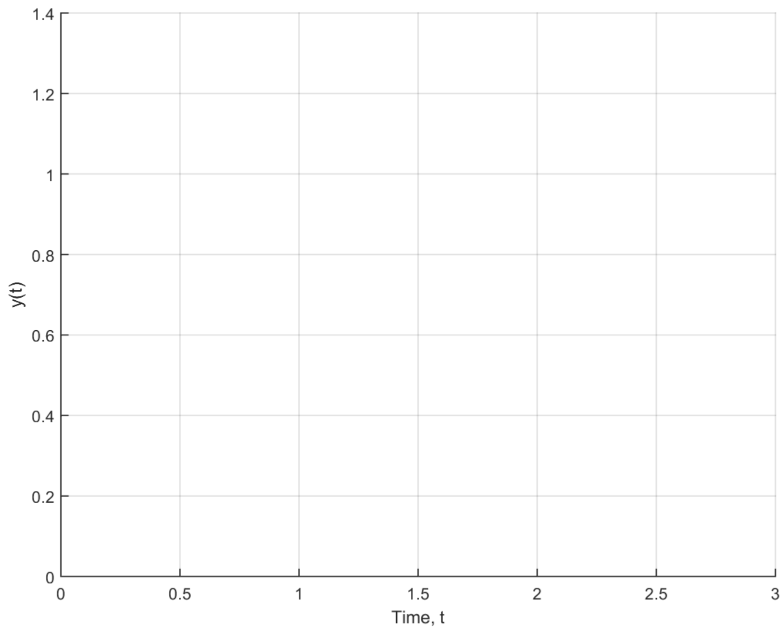
(e) As a function of K_2, what is the steady-state gain from $d \to y$ in the closed-loop system?

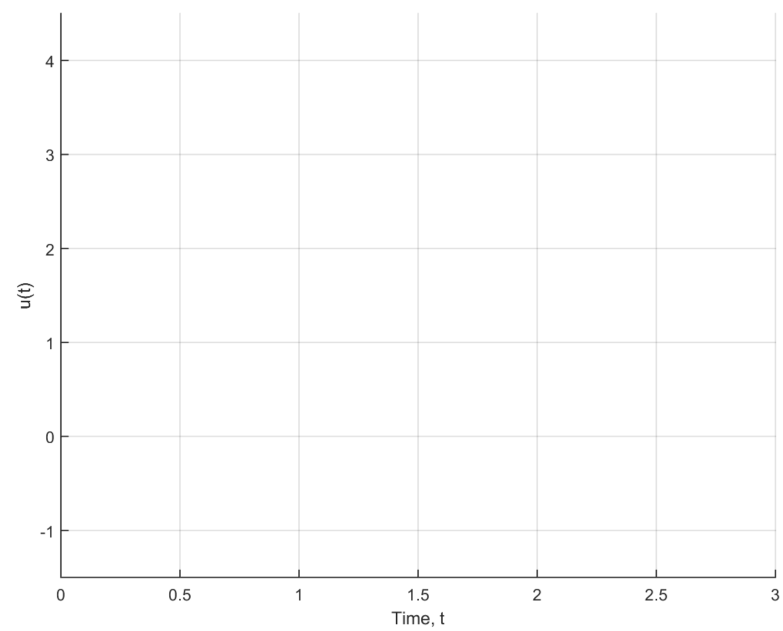
(f) As a function of K_1 and K_2, what is the steady-state gain from $r \to y$ in the closed-loop system?

(g) Choose K_1 and K_2 so that the steady-state gain from $r \to y$ equals 1, and the steady-state gain from $d \to y$ equals 0.1.

(h) With those gains chosen, sketch (try to be accurate) the two responses $y(t)$ and $u(t)$ for the following situation:

$$x(0) = 0, \quad r(t) = \begin{cases} 0 & \text{for } 0 \leq t \leq 1 \\ 1 & \text{for } 1 < t \end{cases}, \quad d(t) = \begin{cases} 0 & \text{for } 0 \leq t \leq 2 \\ 1 & \text{for } 2 < t \end{cases}$$





5. Consider our standard feedback interconnection, consisting of a 1st-order linear controller and proportional plant described by

$$\text{controller}: \quad \begin{aligned} \dot{x}(t) &= Ax(t) + B_1 r(t) + B_2 y_m(t) \\ u(t) &= Cx(t) + D_1 r(t) \end{aligned}$$

$$\text{plant}: \quad y(t) = \alpha u(t) + \beta d(t)$$

The measurement equation is $y_m(t) = y(t) + n(t)$.

(a) Under what conditions (on the parameters $A, B_1, \ldots, D_1, \alpha, \beta$) is the closed-loop system stable? Carefully justify your answer.

(b) Is it possible that the controller, as a system by itself, is unstable, but the closed-loop system is stable. If your answer is "No", please give a careful explanation. If your answer is "Yes", please give a concrete example.

(c) Suppose $\alpha = \beta = 1$. Design the controller parameters so that

- the closed-loop is stable, with specified time-constant, $\tau_{\text{desired}} = 0.2$.
- The steady-state gain from $r \to y$ is 1, even if α, β change by modest amounts (but do not change sign) **after** the controller has been designed and implemented?
- The steady-state gain from $d \to y$ is 0, even if α, β change by modest amounts (but do not change sign) **after** the controller has been designed and implemented?

Show your work, and clearly mark your answers.

(d) In the design above, the two steady-state gains ($r \to y$ and $d \to y$) are completely insensitive to modest changes in α and β. What are some important closed-loop properties that <u>do</u> change if α and β vary?