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UNIVERSITY OF CALIFORNIA, BERKELEY
MECHANICAL ENGINEERING
ME106 Fluid Mechanics
2nd Test, S18 *Prof S. Morris*

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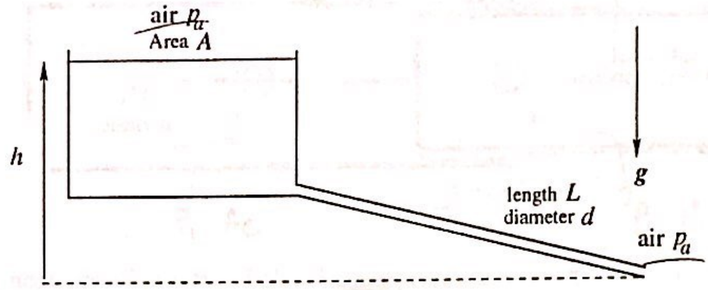
1.(100) The large reservoir drains through a pipe of length L and diameter d within which the power loss is given by $\frac{1}{2}\dot{m} f V^2 \frac{L}{d}$. Derive the differential equation giving dh/dt in terms of h and the constants shown in the figure. (You are not asked to solve the differential equation.)



energy balance

$$\dot{m}\left[\tfrac{1}{2}v^2 + \tfrac{p}{\rho} + gz\right]_1^2 = \cancel{S.P} - P.L \to \tfrac{1}{2}\dot{m} f v^2 \frac{L}{d}$$

$$\cancel{\dot{m}}\left[\tfrac{1}{2}v^2 - gh\right] = -\tfrac{1}{2}\cancel{\dot{m}} f v^2 \frac{L}{d}$$

assuming $\frac{dh}{dt}$ is much smaller than V and do not affect the eqn

assuming : $A \gg \frac{\pi d^2}{4}$

P_a acting at both end

$$\tfrac{1}{2}v^2\left[1 + f\frac{L}{d}\right] = gh$$

$$v = \sqrt{\frac{2gh}{(1+f\frac{L}{d})}} \to ①$$

Mass balance

$$\oint v \cdot \hat{n}\, ds = 0 \quad \text{gen eqn}$$

$$\frac{dh}{dt}\cdot A + \frac{\pi d^2}{4}\cdot V = 0$$

$$\frac{\pi d^2}{4} V = -\frac{dh}{dt}\cdot A$$

$$V = -\frac{dh}{dt}\cdot\frac{4A}{\pi d^2} \to ②$$

good M.B.
+30

① x ②

$$-\frac{dh}{dt}\cdot\frac{4A}{\pi d^2} = \sqrt{\frac{2gh}{(1+f\frac{L}{d})}}$$

$$\boxed{\frac{dh}{dt} = -\frac{\pi d^2}{4A}\sqrt{\frac{2gh}{(1+f\frac{L}{d})}}}$$

1	100
2	100
	200

:)

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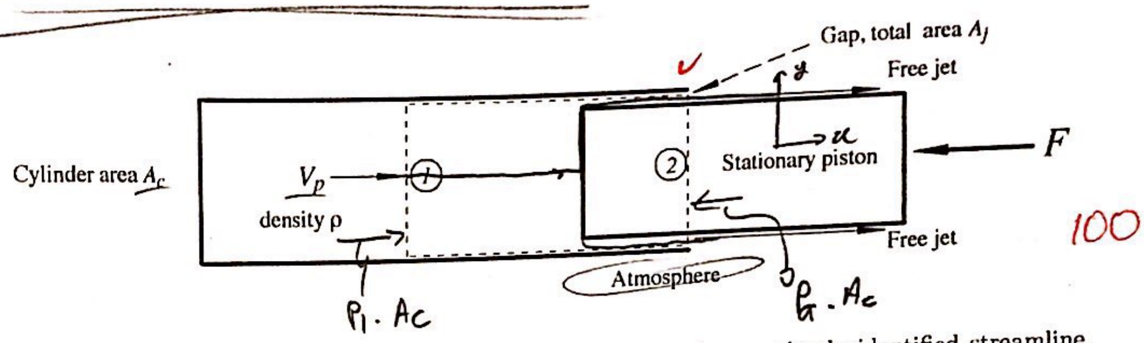
2.(100) In a hydraulic buffer, the force F applied to the buffer piston is balanced by the pressure force exerted on the piston by the fluid. In the figure, the axes are taken to be fixed in the piston. Fluid moves towards the stationary piston with speed V_p, and then leaves the cylinder as a free jet. The flow is quasi–steady, incompressible, and effectively inviscid.



100

(a) By using the incompressible form of the Bernoulli equation along a clearly identified streamline, find the pressure p_1 acting at face 1 of the control volume in terms of atmospheric pressure p_a, ρ, V_p and the unknown velocity V_j in the free jet.

(b) By balancing mass and momentum on the contents of the control volume shown in the figure, and using the result from part (a), find F in terms of ρ, A_c, V_p and A_j. Atmospheric pressure p_a acts equally on all faces of the piston.

a) $\frac{1}{2}\rho V_p^2 + P_1 = \frac{1}{2}\rho V_j^2 + P_a$ BE +20

$$\boxed{P_1 = \frac{1}{2}\rho(V_j^2 - V_p^2) + P_a}$$

$P_1 - P_a = \frac{1}{2}\rho(V_j^2 - V_p^2)$ —(1)

b) Mass balance

$V \cdot \hat{n} = 0$

$-V_p \cdot A_c + V_j A_j = 0$

$V_j A_j = V_p A_c$

$V_p = \frac{V_j \cdot A_j}{A_c}$ +20 MB

$V_j = \frac{V_p \cdot A_c}{A_j}$

Momentum balance

$\oint \rho \underline{v} \cdot (\underline{v} \cdot \hat{n})\, ds = F_{total}$ (x-direction)

gen eqn +10

$\rho V_p(-V_p)A_c + \rho V_j(V_j)A_j = P_1 \cdot A_c - P_a \cdot A_c - F$

mom flux +15   Fext +20

$F = (P_1 - P_a)A_c + \rho V_p^2 A_c - \rho V_j^2 A_j$ —(2)

Continue at right side

(1) X (2)

$F = \frac{1}{2}\rho(V_j^2 - V_p^2)A_c + \rho V_p^2 A_c - \rho V_j^2 A_j$

$F = \frac{1}{2}\rho V_j^2 A_c - \frac{1}{2}\rho V_p^2 A_c + \rho V_p^2 A_c - \rho V_j^2 A_j$

$F = \frac{1}{2}\rho A_c V_p^2 \cdot \frac{A_c^2}{A_j^2} - \frac{1}{2}\rho V_p^2 A_c + \rho V_p^2 A_c - \rho V_p^2 \frac{A_c^2}{A_j^2} \cdot A_j$

$F = \frac{1}{2}\rho V_p^2 \cdot A_c\left(\frac{A_c^2}{A_j^2} - 1 + 2 - 2\frac{A_c}{A_j}\right)$

$F = \frac{1}{2}\rho V_p^2 \cdot A_c\left(\frac{A_c^2}{A_j^2} - 2\frac{A_c}{A_j} + 1\right)$

$$\boxed{F = \frac{1}{2}\rho V_p^2 A_c\left(\frac{A_c}{A_j} - 1\right)^2}$$ +15

∴ F always positive (direction of F correct) make sense