## Physics 7A (Prof. Hallatschek)

## Second Midterm, Fall 2017, Berkeley, CA

Rules: This midterm is closed book and closed notes. You are allowed two sides of one-half sheet of 8.5 " x 11 " of paper on which you can whatever note you wish. You are not allowed to use scientific calculators. Cell phones must be turned off during the exam, and placed in your backpacks. In particular, cell-phone-based calculators cannot be used.

## Please make sure that you do the following during the midterm:

- Write your name, discussion number, ID number on all documents you hand in.
- Make sure that the grader knows what s/he should grade by circling your final answer.
- Answer all questions that require a numerical answer to three significant figures.

We will give partial credit on this midterm, so if you are not altogether sure how to do a problem, or if you do not have time to complete a problem, be sure to write down as much information as you can on the problem. This includes any or all of the following: Drawing a clear diagram of the problem, telling us how you would do the problem if you had the time, telling us why you believe (in terms of physics) the answer you got to a problem is incorrect, and telling us how you would mathematically solve an equation or set of equations once the physics is given and the equations have been derived. Don't get too bogged down in the mathematics; we are looking to see how much physics you know, not how well you can solve math problems.

If at any point in the exam you have any problems, just raise your hand, and we will see if we are able to answer it.

Disc Sec Number:
Name: $\qquad$
$\qquad$
Disc Sec GSI:
Signature: $\qquad$
$\qquad$
Student ID Number: $\qquad$

| Problem | Possible | Score |
| :---: | :---: | :---: |
| 1 | 20 |  |
| 2 | 20 |  |
| 3 | 20 |  |
| 4 | 20 |  |
| 5 | 20 |  |
| Total | 100 |  |

NOTE: Unless otherwise stated, your answers may contain any symbols defined in the problem statement and any physical constants such as the acceleration due to gravity g.

## Problem 1 (20 points)

A block of mass $M$ is suspended from the ceiling at a point A by a massless cord of length L. A bullet with mass $m$ is fired into the block with a horizontal velocity $v$. Treat the bullet and the block as point masses. Neglect air resistance and the friction of the pivot point.

Assume first that the bullet collides with the block completely inelastically (meaning that they stick together).
(a) The cord-block-bullet system then rotates about a fixed axis at point $A$. What is the angular velocity $\omega_{0}$ of the block right after the
 collision?
(b) After the collision, what is the maximum angle $\theta_{\max }$ the cord-block-bullet system rotates counterclockwise before it stops and starts moving back towards the opposite direction?

Now assume that the bullet collides with the block elastically (meaning that bullet and block fly apart after the collision). Find the maximum angle $\theta_{\max }$ of the box-cord system for the special cases that
(c) the mass of the bullet is equal to the mass of the block ( $m=M$ )
(d) the mass of the bullet is much larger than the mass of the block ( $m \gg M$ )

Problem 2 (20 points) Gravitational pull from ring-like structures
The following problem is inspired by the fact that several planets (Jupiter, Saturn, Uranus) are encircled by rings, perhaps composed of material that failed to form a satellite. In addition, many galaxies contain ring-like structures.

Consider a homogeneous thin ring (e.g. assume no thickness) of mass $M$ and of radius $r$.
(a) Find the direction and magnitude of the gravitational attraction force the ring exerts on a particle of mass $m$ located on the ring's central axis a distance $x$ from the ring center.
(b) Suppose the particle falls from rest as a result of the attraction of the ring. What is the speed with which the particle passes through the center of the ring?
(c) What initial velocity is needed at least for the particle to escape the gravitational field of the ring from its initial position at $x$ ?

## Problem 3 (20 points)

A ball is attached to a horizontal massless cord of length $\ell$ whose other end is fixed, see Figure. A thin peg is located a distance $h$ directly below the point of attachment of the cord. The ball is released with an initial downward velocity $v_{0}$. Ignore any frictional forces.
(a) What is its speed at the lowest point of its path?
(b) What is the tension just after the cord strikes the peg?
(c) What condition must $h$ satisfy so that the ball describes a full circle around the peg?
(d) Given this condition is satisfied, what is the tension in the cord at the highest point of the circle.


## Problem 4. Trebuchet (20 points)

A trebuchet is a type of historical siege engine which uses a swinging arm to throw a projectile. Consider the attached simple design of a trebuchet: A beam fixed about a hinge has two masses at its ends. The projectile is a ball of mass $m$ and is placed in a basket at the left end of a beam. A heavy counter weight $M$ is suspended from the right end of the beam. The beam is free to rotate around a hinge, which is fixed in space.

To launch the trebuchet, the counter weight is allowed to swing down, which accelerates the projectile at the other end via the clockwise rotation of the connecting beam. The rotation of the beam is suddenly stopped (by pegs) when the angle between beam and horizontal reaches 45 degrees. The projectile then flies out of the basket towards the enemy.

Suppose you are an engineer and would like to optimize the efficiency of the trebuchet. Assume that the distance $l$ between hinge and counter weight is fixed, as well as the masses. The length $L$, however, can be chosen arbitrarily.
(a) What is the maximal value of $L$ for which the trebuchet works, and why?
(b) Find the velocity of the projectile when it leaves the basket. Assume that the beam is weightless. Treat the projectile and the counter weight as point masses.
(c) Repeat problem (b) assuming that the beam has constant mass density. Recall that the moment of inertia of a beam of mass $m_{B}$ and length $R$ rotating about its center of mass is $m_{B} R^{2} / 12$.


Problem 5. Rolling without slipping ( 20 points)
Figure (a) below shows a disk of mass $M$ and radius $R$ on a table. A rod with negligible mass is attached to the disk at a distance, $l$, from the center of the disk, and an angle, $\theta$, from the vertical. This rod is oriented perpendicular to the plane of the disk, while parallel to the table; the disk can rotate freely about this rod. Recall that the moment of inertia of a disk of mass $M$ and radius $R$ rotating about its center of mass is $M R^{2} / 2$.
(a) A horizontal force $F$ is applied to the disk at the rod. If the disk rolls without slipping, what is the static friction, $f_{s}$, between the disk and the table?
(b) Suppose the force $F$ is applied at the same position but, now, at a positive angle $\alpha<90^{\circ}$ from the horizontal, as indicated in Figure (b). Find the maximal angle $\alpha_{c}$ for which the disk rolls to the right (i.e. the disk rolls to the right if $\alpha<\alpha_{c}$; the disk rolls to the left if $\alpha>\alpha_{c}$ ). Assume that the force $F$ is unable to lift the disk, and that the disk rolls without slipping.


