NAME: **STUDENT ID:**

Math 54 , first midterm, Fall 2018, John Lott

This is a closed book, closed notes, closed calculator, closed computer, closed phone, closed network, open mind exam.

Be sure to write your name and student id number on the top of EVERY page that you turn in.

Name of neighbor to your left ______________________________

Name of neighbor to your right ______________________________

Write your answers in the boxes provided. For full credit, show your work, too, (except for the true/false questions) and cross out work that you do not want us to grade.

You can write on the backs of pages. You can also use the back of this page, and the back of the last page, as scratch paper. If you need additional scratch paper, ask me for it.

NAME: **STUDENT ID:**

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1. (10 pts) Is $\begin{bmatrix} 3 \\ -7 \\ -3 \end{bmatrix}$ in the span of the vectors

$$\begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}, \begin{bmatrix} -4 \\ 3 \\ 8 \end{bmatrix}, \begin{bmatrix} 2 \\ 5 \\ -4 \end{bmatrix}?$$

Write YES or NO in the box.

Justify your answer.

NO

The augmented matrix is

$$\begin{bmatrix} 1 & -4 & 2 & 3 \\ 0 & 3 & 5 & -7 \\ -2 & 8 & -4 & -3 \end{bmatrix}.$$

Performing a row operation gives

$$\begin{bmatrix} 1 & -4 & 2 & 3 \\ 0 & 3 & 5 & -7 \\ 0 & 0 & 0 & 3 \end{bmatrix}.$$

The last row is inconsistent.

NAME: **STUDENT ID:**

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2. (10 pts) (No partial credit, but show your work.)

Write the inverse of

$$\begin{bmatrix} 1 & 2 & 4 \\ 1 & 3 & 5 \\ 1 & 2 & 3 \end{bmatrix}$$

in the box.

$$\boxed{\phantom{\begin{matrix} \\ \\ \\ \\ \end{matrix}}}$$

The answer is

$$\begin{bmatrix} 1 & -2 & 2 \\ -2 & 1 & 1 \\ 1 & 0 & -1 \end{bmatrix}.$$

NAME: **STUDENT ID:**

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3. (10 pts) A linear transformation T from $\mathbb{R}^2$ to $\mathbb{R}^3$ satisfies

$$T\left(\begin{bmatrix}1\\1\end{bmatrix}\right) = \begin{bmatrix}4\\0\\0\end{bmatrix}, \quad T\left(\begin{bmatrix}1\\-1\end{bmatrix}\right) = \begin{bmatrix}0\\6\\0\end{bmatrix}.$$

Write the standard matrix of T in the box. Justify your answer.

$$\begin{bmatrix}2 & 2\\3 & -3\\0 & 0\end{bmatrix}.$$

This is because

$$T\left(\begin{bmatrix}1\\0\end{bmatrix}\right) = \begin{bmatrix}2\\3\\0\end{bmatrix}, \quad T\left(\begin{bmatrix}0\\1\end{bmatrix}\right) = \begin{bmatrix}2\\-3\\0\end{bmatrix}.$$

NAME: **STUDENT ID:**

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4. (12 pts) Suppose that B is an invertible square matrix with the property that for both B and B^{-1}, all of their entries are integers. Show that $\det(B)$ is 1 or -1.

$$\det(B)\det(B^{-1}) = \det(I) = 1.$$

Since B and B^{-1} both have integer entries, $\det(B)$ and $\det(B^{-1})$ are both integers, so $\det(B)$ must be ± 1.

NAME: **STUDENT ID:**

NAME: **STUDENT ID:**

5. (18 pts) Check only one box per question. No explanation is necessary.
WARNING: there are multiple versions of this problem. If you copy off of a neighbor, you will probably get around half of the answers wrong.

If $\{\mathbf{x_1}, \mathbf{x_2}, \mathbf{x_3}, \mathbf{x_4}\}$ is a set of linearly independent vectors in $\mathbb{R}^4$ then $\{\mathbf{x_1}, \mathbf{x_2}, \mathbf{x_3}\}$ is linearly independent.

☐ <u>TRUE</u> ☐ FALSE

If A is an $m \times n$ matrix then the null space of A is a subspace of $\mathbb{R}^m$.

☐ TRUE ☐ <u>FALSE</u>

If A is a matrix and $A^5 = I$ then A is invertible.

☐ <u>TRUE</u> ☐ FALSE

If A and B are $n \times n$ matrices, and AB is invertible, then A and B are invertible.

☐ <u>TRUE</u> ☐ FALSE

If a matrix A has linearly dependent columns then $A\mathbf{x} = \mathbf{0}$ has infinitely many solutions.

☐ <u>TRUE</u> ☐ FALSE

NAME: **STUDENT ID:**

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The set of vectors $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ so that $x_1 + x_2 + x_3 \geq 0$ forms a subspace of $\mathbb{R}^3$.

☐ TRUE ☐ <u>FALSE</u>

If A is a 5×5 matrix such that $\det(2A) = \det(A)$ then $A = 0$.

☐ TRUE ☐ <u>FALSE</u>

If T is a one-to-one linear transformation from $\mathbb{R}^n$ to $\mathbb{R}^n$ then T is onto.

☐ <u>TRUE</u> ☐ FALSE

If V is a vector space, and H_1 and H_2 are subspaces, then the union of H_1 and H_2 (i.e. the set of vectors that lie in H_1 *or* H_2) is always a subspace.

☐ TRUE ☐ <u>FALSE</u>

NAME: **STUDENT ID:**