$\qquad$ Discussion Section \# $\qquad$
Signature $\qquad$ Discussion Section GSI $\qquad$
Student ID\# $\qquad$
This exam is closed book, but you are allowed one 8.5 " x 11 " (double-sided) page of handwritten notes. You may use a calculator, however NO wireless calculators are allowed. Anyone using a wireless calculator will forfeit their exam and automatically receive the score of zero. You may use $g=10 \mathrm{~m} / \mathrm{s}^{2}$.
a) Write your name, Discussion Section \#, GSI name and SID\# on the top of all materials you intend to hand in and want to be graded.
b) Remember to circle all of your final answers.
c) Express all numerical results to 3 significant figures. Cross out any work you decide is incorrect, with an explanation in the margin.

Read through the entire exam to start. Work to maximize your credit - try to obtain at least partial credit on every part of every problem. Do your work clearly so we can easily follow. Show all work, using the front and back sides of this exam paper. If you do not show relevant work for any part of a problem, you will not be awarded any credit, even if the answer is correct. If you recognize that an answer does not make physical sense and you do not have time to find your error, write that you know that the answer cannot be correct and explain how you know this to be true. (We will award some credit for recognizing there is an error.) Do not get bogged down in algebra - if you have enough equations to solve for your unknowns, box the equations, state how you would finish, and move on (you can go back and complete the algebra later if you have time). And if you have questions about the interpretation of a problem, please ask!

| Problem 1 |  |
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| $\mathbf{9}$ |  |
| TOTAL |  |

Total of 200 points (points for each part indicated in problem).

1. Block A (with mass $m_{\mathrm{A}}$ ) slides on block B (with mass $m_{\mathrm{B}}$ ), which slides on an inclined plane, as shown in the drawing below. The coefficient of kinetic friction between the inclined plane and block B is $\mu$, but there's no friction between block A and block B . The string and pulley are massless. Also, we know that $m_{\mathrm{B}}>m_{\mathrm{A}}$.
a) 5 Points. Draw a separate force diagram for block A and block B,
 showing all the forces.
b) 10 Points. Assuming the blocks start from rest at $t=0$, find the speed of block B down the inclined plane at arbitrary time $t$ before block A slides off completely. Express your answer in terms of $m_{\mathrm{A}}$, $m_{\mathrm{B}}, \theta, \mu, g$, and $t$.
c) 5 Points. Find the tension in the string connecting the blocks.
2. A ball of mass $M=2 \mathrm{~kg}$ at the end of a string of length $R=2 \mathrm{~m}$ revolves in a vertical circle as shown. The motion is circular but not uniform because of the force of gravity.
a) 5 Points. Determine the minimum speed the ball must have at the highest point in the circle so the string doesn't slacken.
b) 10 Points. Determine the direction and magnitude of the tangential acceleration, the radial acceleration, and the tension in the string at $\theta=30^{\circ}$ below the
 horizontal if the ball's speed is $6 \mathrm{~m} / \mathrm{s}$.
c) 5 Points. If the string breaks at $\theta=30^{\circ}$ at $t=0$, find the subsequent trajectory $y(t), x(t)$ of the ball where $x=0$ and $y=0$ at $t=0$.
3. A solid cylinder of length $L$ and radius $R$ has a mass $M$. Two cords are wrapped around the cylinder, one near each end, and the cord ends are attached to hooks on the ceiling. The cylinder is held horizontally with the two cords exactly vertical and is then released. Give your answers in terms of $L, R, M$, and $g$.
a) 15 Points. Find the tension in each cord as they unwind without slipping. (You
 must show your derivation of the moment of inertia to get full credit.)
b) 5 Points. Find the linear acceleration of the cylinder as it falls.
4. Two particles, each of mass $M$ and speed $v$, move as shown. They simultaneously strike the ends of a uniform rod of mass $M$ and length $d$ which is pivoted at its center. The particles stick to the ends of the rod.
a) 5 Points. Find the magnitude and direction of the angular momentum with respect to the center of the rod before the collision.
b) 5 Points. What is the angular momentum after the collision? Justify your answer.
c) 10 Points. Find the angular speed of rotation of the particles and the rod after the
 collision.
d) 5 Points. How much kinetic energy is lost in the collision of the two particles with the rod? Where does the energy go?
5. Suppose the Sun is traveling with velocity $220 \mathrm{~km} / \mathrm{s}$ in a circular orbit of radius $2.5 \times 10^{17} \mathrm{~km}$ about the center of our galaxy. $\left(G=6.67 \times 10^{-11} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{kg}^{2}\right)$
a) 10 Points. If the mass distribution of the galaxy is spherically symmetric about the center, find the total galactic mass contained inside the orbit of the Sun.
b) 10 Points. Assuming there is no additional mass outside the orbit of the Sun, derive how fast an object at that distance would have to travel to escape from the galaxy.
6. A bicyclist traveling at speed $v$ makes an emergency stop by applying brakes on both wheels and going into a controlled skid. The center of mass of the bicycle plus rider (total mass $M$ ) is at height $h$ from the pavement and at a horizontal point equidistant from the two wheels.
a) 15 Points. If the coefficient of kinetic friction between the wheels and the pavement is $\mu_{\mathrm{k}}$, find the normal and frictional forces exerted by the pavement on the front wheel and on the back wheel.

b) 10 Points. Find the relationship between $h$ and $L$, the distance between the centers of the wheels, to keep the bicycle from overturning and throwing the rider over the handlebars.
7. A water "rocket" consists of a cubic chamber of side length $L=0.1 \mathrm{~m}$ that is filled with water (neglect the mass of the chamber walls relative to the water). The lid (also of negligible mass) is held on with two (stiff) springs of spring constant $k=10^{7} \mathrm{~N} / \mathrm{m}$, which are stretched a distance 0.05 m from their equilibrium extension. A small circular hole of radius $R$ is on the bottom of the box through which the water can escape. ( 1 atmosphere $=10^{5} \mathrm{~Pa}$. Density of water $=10^{3} \mathrm{~kg} / \mathrm{m}^{3}$ )
a) 5 Points. What is the pressure just under the lid of the box?
b) 10 Points. What is the velocity of the liquid leaving the box?

c) 10 Points. How large must the radius $R$ of the hole be for the water rocket to just get off the ground? (Hint: The flow of momentum out of the box is $d P / d t=d(m v) / d t=v d m / d t$, where $v$ is the velocity of the escaping water and $d m / d t$ is the mass of water per unit time leaving the box.)
8. 20 Points. A block of mass $M$ is attached to a rigid support by a spring of force constant $k$, and it is executing simple harmonic motion with amplitude $x_{0}$ on a horizontal, frictionless table. A bullet of mass $m$ and speed $v$ strikes the block when the block passes the equilibrium $(x=0)$ point, going the same direction as the bullet, as shown. The bullet remains embedded in the block. Determine the amplitude and frequency of the
 resulting simple harmonic motion, in terms of $m, M, v, x_{0}$ and $k$.
9. One end $(x=0)$ of a rope of mass per unit length, $\mu$, is moved up and down in simple harmonic motion $y=y_{\mathrm{o}} \sin \omega t$.
a) 6 Points. If the rope is under tension $F_{T}$, find the
 wavelength, frequency, and phase speed in terms of the given quantities.
b) 4 Points. Find the function of $x$ and $t$ that describes the wave that results.
c) 5 Points. Find the total mechanical energy (potential plus kinetic) per unit length, and the power carried by the wave past a fixed point.
d) 10 Points. Suppose the rope can be fixed to $y=0$ at $x=L$. Find an expression for the values of $L$ for which standing waves are set up.
