Please write your answers with pen or dark pencil. Answers will only be graded within the boxes below each question. To receive full credit, please write the equation that you use to answer the question, and then give your answers in numerical values.

You are given the following effective potential for an electron orbiting around the nucleus of the H-atom with angular momentum \( l \) and radial velocity \( v_r \):

\[
E = \frac{mv_r^2}{2} + \frac{l^2}{2mr^2} - \frac{q^2}{4\pi\varepsilon_0 r}
\]

The radii of the circular orbits of this effective potential are:

\[
r_0 = n^2(a_0)
\]

where \( a_0 = 0.5 \times 10^{-10} \) m and \( n \) is Bohr’s quantization of angular momentum (\( l \)).

The energy levels of Bohr’s model are given by, where \( R_H = 13 \) eV:

\[
E_n = -\frac{R_H}{n^2}
\]

To simplify the calculations, use the following constants:

- \( c = 3 \times 10^8 \) m/s (speed of light)
- \( m = 9 \times 10^{-31} \) kg (mass of electron)
- \( h = 6 \times 10^{-34} \) J-s or \( 3.8 \times 10^{-15} \) eV-s (planck’s constant \( h \))
- \( \hbar = 1 \times 10^{-34} \) J-s or \( 0.6 \times 10^{-15} \) eV-s (\( h/2\pi \))

To receive full credit, please write the equation that you use to answer the question, and then give your answers in numerical values.
1. (10 pts) For a particular metal surface, radiation of intensity 1.0 W/cm² (W= Joules/s) and wavelength 250 nm yields electrons of kinetic energy 0.5 eV.

(a) What is the work function (in eV) of this material?

\[ KE = h \cdot \nu - \text{wf} \]
\[ \nu = c \]
\[ v = \frac{3 \times 10^8 \text{ m/s}}{250 \times 10^{-9} \text{ m}} \]
\[ \nu = 1.2 \times 10^15 \]
\[ 0.5 \text{ eV} = 3.8 \times 10^{-15} \text{ eV s} \cdot 1.2 \times 10^15 \text{ Hz} - \text{wf} \]
\[ 0.5 \text{ eV} = 4.56 \text{ eV} - \text{wf} \]
\[ \text{wf} = 4.06 \text{ eV} = 4.1 \text{ eV} \]

(b) If you double the intensity of the radiation, what will be the effect on the electron kinetic energy?

There will be no effect on the electron's kinetic energy, since the KE of the electrons is only dependent on the radiation's frequency (and wavelength).

(c) If you do the same experiment with a different metal, and you don’t get any electrons coming out, what can you conclude about the second metal’s work function?

The second metal’s work function is higher, since when it absorbs the same amount of energy, the electrons do not escape, indicating they don’t have enough energy to break free of their binding energy.

* in magnitude
2. (20 pts) Consider the effective coulomb potential energy diagram (see Figure) that describes electrons orbiting a nucleus. Use it to answer the following questions, some of which will require you to draw on the figure.

Figure 1: Effective Coulomb potential energy $V_{\text{eff}}(r)$ vs radial distance $r$.

(a) Above what energy are the electrons not bound to the nucleus (unbound)?

0 J.

(b) Draw about 5 horizontal energy levels corresponding to the bound electrons, showing how the energy level spacing changes. Use a horizontal line to identify both the energy and the spatial region to which the electrons are bound.

(c) Locate the equilibrium circular orbit, or Bohr’s radius, on the horizontal axis.
(d) If the radius is $4a_0$, what is the quantized angular momentum in Bohr's model that describes this effective potential energy?

\[
\frac{1}{2}\hbar^2 \frac{d^2}{dr^2} + V_\text{eff} = E_n = n^2 \frac{\hbar^2}{2m}
\]

When $r = 2a_0$, $n = 2$:

\[
\begin{align*}
4a_0 &= 2a_0 \\
4 &= n^2 \\
4a_0 &= n^2 a_0 \\
\hbar &= n \hbar \\
\lambda &= n\hbar \\
\lambda &= 2 \cdot 1 \times 10^{-34} \text{ J} \cdot \text{s}
\end{align*}
\]

(e) Using the classical description of angular momentum, find the velocity of the electron in the circular orbit. Use this to estimate the wavelength of the electron.

\[
l = m \cdot r \cdot v
\]

\[
2 \times 10^{-34} \text{ kg} \cdot \text{m}^2 / \text{s} = 9 \times 10^{-31} \text{ kg} \cdot (4 \times 10^{-10} \text{ m}) \cdot v
\]

\[
v = 1.11 \times 10^6 \text{ m/s}
\]

\[
1.11 \times 10^6 \text{ m/s} = \lambda \cdot v
\]

\[
\lambda = 1 \times 10^{-10} \text{ m}
\]
3. (7 pts) If the wavelength of light emitted by the H-atom is 470 nm, and the electron ends up in the second lowest (n=2) energy level, what energy level must it have come from?

\[ \Delta E = -\frac{\Delta E}{\Delta E} \left( \frac{1}{n_f^2} - \frac{1}{n_i^2} \right) \]

\[ \Delta E = h \nu = \frac{h \cdot c}{\lambda} = \frac{3.8 \times 10^{-15} \text{ J} \cdot \text{s} \cdot \frac{3 \times 8 \times 10^8}{470 \times 10^{-9} \text{ m}}}{2.425 \text{ eV}} = 2.425 \text{ eV} \]

\[ 2.425 \text{ eV} = 13 \text{ eV} \left( \frac{1}{n_f^2} - \frac{1}{4} \right) \]

\[ -0.1865 = \frac{1}{n_i^2} - \frac{1}{4} \]

\[ -0.0635 = \frac{1}{n_i^2} \]

\[ n_i = \sqrt{\frac{1}{0.0635}} \]

\[ n_i = \sqrt{16} \]

\[ n_i = 4 \]
4. (24 pts) Consider that the electron can have one of the following types of waves, either travelling or standing. In the region where the electron exists, the potential energy is zero. You only need to give symbolic, not numeric answers.

1. $\sin(kx - \omega t)$

where $k$ varies continuously. $k$ and $\omega$ can be used as constants in answering the questions below.

The following are only defined for $0 < x < L$ and the wave is zero otherwise:

2. $\sin(k_1 x) \cos(\omega_1 t)$

3. $\sin(k_2 x) \cos(\omega_2 t)$

where $k_n$ are discrete values that take on $k_n = n2\pi/L$, which can be used in answering the questions below.

(a) Which one(s) represent a travelling wave? Give its velocity, in terms of the given constants.

$$v = \frac{\lambda}{\omega}$$

$sin(kx - \omega t)$ is a travelling wave

$v = \frac{2\pi}{\omega}$

$w = \frac{2\pi}{\nu}$

$v = \frac{4\pi^2}{k\omega}$
(b) For those that represent standing waves, draw their spatial wave pattern. Include the appropriate number of nodes for each pattern.

3) 1 node, since at $\frac{L}{2}$ the wave is at 0.

(c) Of those that represent bound electrons, which has the higher energy?

3. It is at a higher energy level (in terms of magnitude), so since $E = \frac{\hbar^2 \pi^2 n^2}{2mL^2}$ and all but $n$ are constants, $E \propto n$
(d) If the electron described by a travelling wave is incident upon a double slit, what should the distance between the slits be to observe a well-defined diffraction pattern?

\[
\frac{d}{\lambda} \approx \frac{2\pi}{k} \quad \text{for visible diffraction}
\]

\[
\lambda = \frac{2\pi}{k}
\]

The distance must be similar to \( \frac{2\pi \nu}{k} \) for the diffraction to be observable.

(e) If you increase the energy of that travelling wave, would you increase or decrease the distance between the slits?

- Higher energy = higher frequency \((E = h\nu)\) \(\nu = \text{frequency}\)
- Higher frequency = lower wavelength \((\nu = \frac{1}{\lambda})\)
- Lower wavelength = smaller distance since distance must be near wavelength in size

(f) What does the resulting diffraction pattern of the electron represent quantum mechanically? (Give your answer as a one sentence explanation).

The diffraction pattern proves that electrons can behave as both a wave and a particle since they diffract like light but have a mass that can be detected so as to measure the diffraction.
5. (6 pts) Which of the following is true of the cathode ray experiment?
(a) It showed electrons exist by applying a voltage across electrodes
(b) In the experiment, particles were ejected from the anode (positive electrode)
(c) It measured the charge of the electron to be \(1.6 \times 10^{-19}\) Coulombs
(d) It showed that electrons exhibit wave-like properties

6. (6 pts) Which of the following is true of the Rutherford experiment?
(a) It showed that the positive charge in matter is concentrated in a dense nucleus
(b) It led to the planetary model of the atom
(c) It is described by a potential energy in which positively charged particles can be bound to a region of space
(d) It concluded that the jellium model of matter, with an evenly distributed positive charge and where the electrons were not moving, is correct

7. (6 pts) Which of the following is true of the Milliken experiment?
(a) It used an electric field to balance the gravitational force of charged oil drops.
(b) All of the oil drops were found to "float" at a given applied electric field.
(c) It measured the mass of the electron
(d) It measured the charge of the electron to be \(1.6 \times 10^{-19}\) Coulombs