University of California at Berkeley Department of Physics Physics 7A, Spring 2014

Midterm 2 April 8, 2014

You will be given 120 minutes to work this exam. No books are allowed, but you may use a double-sided, handwritten formula sheet no larger than an 8 ¹/₂" by 11" sheet of paper. No electronics that can send or receive wirelessly are allowed, but you may use a calculator if you want (won't help much...). Your description of the physics involved in a problem is worth significantly more than any numerical answer. Show all work, be careful with signs, and take particular care to explain what you are doing. Please express your answers using the symbols provided in the problem descriptions or define any new symbols you use, tell us why you're writing any new equations, and clearly label any drawings that you make. Write your answers in a blue book (or green book), and do not use any extra scratch paper. Please BOX your answers and use a pencil or a blue or black pen. Good luck!

1) (20 points) Steamroller on the loose.

Jack forgot to set the emergency break when he stepped out of his steamroller for his lunch break. At some point the steamroller started to roll down a smooth incline that made an angle of γ with respect to the horizontal, as shown in the diagram. The mass of the body of the steamroller (everything except for the wheels) is *M*. The front wheel consists of a *solid* cylinder of uniform mass density with a radius *r*, width *D*, and mass 6*M*. The back wheel consists of a single *hollow* cylinder of radius R = 1.5r, width *D*, and mass 5*M*. For this entire problem, both cylinders roll along the ground without slipping and assume that both cylinders are free to rotate about their frictionless axes.

- a) What is the mass volume density, ρ , of the *front* wheel? Express your answer in terms of any combination of *r*, *D*, *M*, and/or *g*.
- b) Which of the two cylinders has a larger moment of inertia about its axis of symmetry?
- c) What is the ratio of the kinetic energy of rotation of the front wheel divided by the kinetic energy of rotation of the back wheel at any moment of time once the steamroller starts to move? Express your answer as a function of any combination of r, D, M, γ , and/or g.
- d) How fast *v* is the steamroller moving after the *front* wheel has undergone one full rotation assuming that it started from rest? Express your answer in terms of any combination of *r*, *D*, *M*, γ , and/or *g*.



2) (20 points) Raining in a cart

An empty cart of mass M is initially sitting still when rain starts to fall at an angle of θ with respect to the vertical, as shown in the diagram. You may assume that the wheels on the cart have negligible moments of inertia and rotate freely about their frictionless axes for this entire problem.

- a) If the cart fills up with water at a steady rate of R (in units of kg/s), then what is the mass of water in the cart after rain has been falling for a total time t?
- b) What is the velocity of the cart after rain has been falling for a total time *t*?
- c) Now the cart runs into a second, empty stationary cart of mass M. There is a massless, ideal spring with spring constant k attached to the second cart, as shown in the diagram. What is the speed of the two cart system right when the spring is as compressed as it gets during the collision?
- d) By how much is the spring compressed at this moment of maximal compression during the collision?



3) (20 points) Thin rotating object

A very thin object of mass M, uniform mass density ρ , thickness T, and longest dimension 2L has a moment of inertia about the *z*-axis given by $I_z = 0.2ML^2$ and a moment of inertia about the *x*-axis given by $I_x = 0.3ML^2$ for axis orientations as shown in the left panel of the figure. The center of mass of the object is located at the origin of this coordinate system.

- a) What is the moment of inertia I_y about the y-axis for this object?
- b) If 4 holes of radius *r* are now drilled into this object, so that the center of each hole is a distance of *D* from the center of mass of the object as shown in the right panel of the figure, what is the resulting moment of inertia I_{yNew} about the *y*-axis for the object?
- c) If a force *F* pointing straight down is now applied to a point on the object that is a height *h* above the center of mass and a distance *d* to the right of the center of mass as shown in the right panel of the figure, then what is the magnitude of the resulting torque from this force alone on the object using the *center of mass* of the object as your pivot point (axis of rotation)?
- d) If the object is balanced on the ground as shown in the right figure panel, then what is the minimum coefficient of static friction between the object and the ground for the object to start rolling without slipping right as the force is first applied? You may express your answer in terms of I_{yNew} rather than using the full expression for the moment of inertia that you found in part b).



4) (20 points) Escape velocity from a trinary star system

A Rocket is situated so that it is exactly distance D away from each of three stars in the configuration shown in the diagram, with one star above, one below, and one to the right of the rocket. Each of the three stars have mass M_S and the rocket has mass M_R .

- a) What is the *direction* and *magnitude* of the force of gravity acting on the rocket due to all 3 of the stars? Use the coordinate axes shown to express your answer, with the *x* axis pointing to the right and the *y* axis pointing upwards in the diagram.
- b) What is the *direction* and *magnitude* of the force of gravity acting *on the system composed of the three stars* due to the rocket?
- c) What is the gravitational potential energy of the rocket due to all three stars? Use the convention that the potential energy goes to zero for points that are extremely far away from the 3 star system.
- d) Assuming that the 3 stars do not move very far from their current configuration during the time that the rocket leaves the system, how fast must the rocket be moving at its current location if it is to be able to travel arbitrarily far away without having to burn any more rocket fuel?



5) (20 points) Blocks sliding on ice

Joe slides a block of wood of mass M on a frozen pond so that it has an initial speed of v_o and it is traveling in the direction of the positive x-axis, so that its initial velocity is $(v_o, 0)$, as shown in the diagram. The moving block then collides with two other blocks of wood that were initially sitting motionless on the ice. After the blocks collide, the second block, with mass 2M, has velocity $(0.2 v_o, -0.1 v_o)$, whereas the third block, with mass 3M, has velocity $(0.1 v_o, 0.1 v_o)$. You may assume that the ice provides a frictionless surface.

- a) After all the collisions have taken place, what is the speed of the center of mass of the system consisting of the three blocks? As always, show your work or justify your answer.
- b) After all the collisions have taken place, what is the final velocity of the first block? Please express your answer as a vector.
- c) Were all the collisions elastic? You may assume that none of the blocks are spinning after all the collisions have occurred.
- d) During this process, what was the impulse that the first block imparted to the system consisting of the other two blocks?