Math 54
Fall 2017
Exam 1
9/26/17
Time Limit: 80 Minutes

Name: ______________________

Student ID: ______________________

GSI or Section: ______________________

This exam contains 6 pages (including this cover page) and 7 problems. Problems are printed on both sides of the pages. Enter all requested information on the top of this page.

> This is a closed book exam. No notes or calculators are permitted.
> We will drop your lowest scoring question for you.

You are required to show your work on each problem on this exam. **Mysterious or unsupported answers will not receive full credit**. A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations will receive partial credit.

If you need more space, there are blank pages at the end of the exam. Clearly indicate when you have used these extra pages for solving a problem. However, it will be greatly appreciated by the GSIs when problems are answered in the space provided, and your final answer **must** be written in that space.

Do not write in the table to the right.

Problem	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
Total:	70	

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1. (10 points) Consider the matrix $A = \begin{bmatrix} -1 & 1 & 1 \\ 0 & 2 & 1 \\ 3 & 8 & 3 \end{bmatrix}$.

   (a) (3 points) Compute the determinant of A and explain why A is invertible.

   (b) (4 points) Find A^{-1}.

(c) (3 points) Use your answer from the previous part to solve $A\mathbf{x} = \begin{pmatrix} 1 \\ 2 \\ 9 \end{pmatrix}$.

2. (a) (5 points) Find all values of s such that $\mathbf{b} = \begin{pmatrix} s \\ 2 \\ 2 \\ 1 \end{pmatrix}$ is in the span of $\begin{pmatrix} -1 \\ 0 \\ 1 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 3 \\ 2 \\ 3 \\ 2 \end{pmatrix}$.

(b) (5 points) Determine the values of s for which the linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^4$ corresponding to the matrix below is onto. Then, determine the values of s for which it is one-to-one.

$$\begin{bmatrix} -1 & 3 & s \\ 0 & 2 & 2 \\ 1 & 3 & 2 \\ 1 & 2 & 1 \end{bmatrix}$$

3. (10 points) Label the following statements as True or False. The correct answer is worth 1 point. An additional point will be awarded for a correct brief justification. No points will be awarded if it is not clear whether you intended to mark the statement as True or False.

   (a) (2 points) The equation $A\mathbf{x} = \mathbf{b}$ has a solution if and only if $\mathbf{b}$ is in the span of the columns of A.

   (b) (2 points) If A is an $m \times n$ matrix and $n > m$, then the columns of A are linearly dependent.

   (c) (2 points) If A is an $n \times n$ matrix and $\det(A) = 0$, then the system $A\mathbf{x} = \mathbf{0}$ has a unique solution.

   (d) (2 points) Any two $n \times n$ elementary matrices commute because they correspond to row operations.

   (e) (2 points) If A and B are $n \times n$ matrices and AB is invertible, then A and B are invertible.

4. (10 points) Find all values of a, b, c so that the vector $\begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$ solves the linear system associated to the following augmented matrix.

$$\left[\begin{array}{ccc|c} a+1 & b & 0 & c \\ 0 & c & a & 2 \\ a+b & -1 & -c & 0 \end{array}\right]$$

5. (10 points) For each of the following requirements, provide an example of an $n \times n$ matrix A satisfying those requirements **OR** explain why no such matrix exists. If you are providing an example, you can choose any value of n that you like.

   (a) (3 points) The columns of A span $\mathbb{R}^n$ and are linearly independent.

   (b) (3 points) The columns of A do not span $\mathbb{R}^n$, but are linearly independent.

   (c) (4 points) The columns of A do not span $\mathbb{R}^n$, and are not linearly independent.

6. (10 points) Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation given by multiplication by a 2×2 matrix A. Let S be a subset of $\mathbb{R}^2$.

   (a) (3 points) Write a formula relating the area of S and the area of $T(S)$.

   (b) (4 points) Suppose that T is the linear transformation given by rotation $\pi/2$ radians counterclockwise around the origin, then reflection over the x-axis, and then scaling by 3 in the y-direction. Find A.

   (c) (3 points) If S is the interior of the circle of radius 2 (which has area 4π). Find the area of $T(S)$ where T is as in part (b).

7. (10 points) Consider the two equations below.

$$x_1 + 4x_2 - 5x_3 = 0$$

$$2x_1 - x_2 + 8x_3 = 9$$

(a) (3 points) For each equation, describe its solution set geometrically; for example, your answer could be of the form: a plane, a sphere, etc.

(b) (7 points) Determine if these two sets intersect and if so, describe their intersection geometrically as in part (a).

Extra space.

Extra space.