## Physics 7C, Speliotopoulos <br> Second Midterm, Fall 2015 <br> Berkeley, CA

Rules: This midterm is closed book and closed notes. You are allowed two sides of one sheet of paper on which you may write whatever you wish. Cell phones must be turned off during the exam, and placed in your backpacks..

Please make sure that you do the following during the midterm:

## - Show all your work in your blue book

- Write your name, discussion number, ID number on all documents you hand in.
- Make sure that the grader knows what s/he should grade by circling your final answer.
- Cross out any parts of the your solutions that you do not want the grader to grade.

Each problem is worth 20 points. We will give partial credit on this midterm, so if you are not altogether sure how to do a problem, or if you do not have time to complete a problem, be sure to write down as much information as you can on the problem. This includes any or all of the following: Drawing a clear diagram of the problem, telling us how you would do the problem if you had the time, telling us why you believe (in terms of physics) the answer you got to a problem is incorrect, and telling us how you would mathematically solve an equation or set of equations once the physics is given and the equations have been derived. Don't get too bogged down in the mathematics; we are looking to see how much physics you know, not how well you can solve math problems.

If at any point in the exam you have any questions, just raise your hand, and we will see if we are able to answer them.

Copy and fill in the following information on the front of your bluebook:
Name: $\qquad$ Disc Sec Number: $\qquad$
Signature: $\qquad$ Disc Sec GSI: $\qquad$
Student ID Number: $\qquad$

1. The figure to the right shows a sliver of glass with small angular opening $\theta$ and index of refraction, $n$. White light is incident on the sliver from above. Looking at the glass from above, you notice that light with wavelength, $\lambda$, was particularly bright at a distance, $L$, from the left; no other wavelengths are accentuated. What is $\theta$ ?
 Express it in terms of $n, L$, and $\lambda$.
2. At $t=0$ in his stationary frame, $S$, Tom suddenly sees Dick throw a ball towards him with velocity $-u_{B}=-\frac{3}{5} c$ (see figure on right). Tom catches the ball with his robotic arm at a time, $T$. Harry is in the moving frame, $S^{\prime}$, moving to the right with speed, $v=\frac{4}{5} c$. The origins of $S$ and $S^{\prime}$ are at the same point at $t^{\prime}=t=0$.

a. Where, $x_{0}$, and when, $t_{0}$, was the ball thrown in Tom's frame?
b. Where, $x_{0}^{\prime}$, and when, $t_{0}^{\prime}$, was the ball thrown in Harry's frame? (If you cannot get part a, you can express your answer in terms of $x_{0}$ and/or $t_{0}$ for partial credit.)
c. How long did it take the ball to travel from Dick to Tom in Harry's frame? (If you cannot get part a, you can express your answer in terms of $x_{0}$ and/or $t_{0}$ for partial credit.)
3. The figure on the right shows a star exploding in the stationary frame, $S$, with the remnants of the star expanding in a sphere with speed, $u$. The moving frame, $S^{\prime}$, moves to the right with velocity, $v$; the origins of the two frames are coincident when the star exploded. Consider the bit of mass moving off at an angle, $\theta$.
a. What is $\tan \theta^{\prime}$ where $\theta^{\prime}$ is the angle that this mass makes in $S^{\prime}$ ? Express it in terms of $\theta, c, u$, and $v$.
b. Assume that $u<v$. Find the angle $\theta$ for which $\tan \theta^{\prime}$ is maximum
 and show that at this angle

$$
\left(\tan \theta^{\prime}\right)_{\max }=-\left(1-\frac{v^{2}}{c^{2}}\right)^{1 / 2}\left(1-\frac{u^{2}}{v^{2}}\right)^{-1 / 2} \frac{u}{v}
$$

4. The figure on the right shows a particle with mass, $M$, and momentum, $\vec{p}_{M}$, decaying into two particles, each of mass, $m \ll M / 2$. Find the magnitude of the momentum, $p_{1}$, of one of these particles in terms of $M, p_{M}$, and $\theta$. You can assume that the two decayed particles are ultra-relativistic so that $E_{1} \approx p_{1} c$ and $E_{2} \approx p_{2} c$ to cut down on the algebra. (I would strongly recommend that you use four-momentum methods to solve this problem.)

5. Figure A on the right shows an incident plane wave,

$$
E_{i}=E_{0} \sin (k x-\omega t)
$$

incident on a single slit with width, $D$, placed at a distance, $L$, from a screen. Figure B shows the complement, with the same incident plane wave incident on a slab with the same width, $D$, placed the same distance, $L$, from a screen. Babinet's principle in optics (in the Kirchoff limit) states that the electric field, $E_{C}(\vec{r}, t)$, for the slab in Fig. B is related to the electric field, $E_{S}(\vec{r}, t)$, due to the slit


Figure A


Figure B in Fig. A by

$$
E_{C}(\vec{r}, t)=E_{i}(\vec{r}, t)-E_{S}(\vec{r}, t) .
$$

Use this to find the intensity, $I(\theta)$, of the diffraction pattern of the light on the screen as a function of $\theta$. Express your answer in terms of $\theta, E_{0}, k, L, D$, and any fundamental constants. Remember that the electric field due to the single slit is

$$
E_{s}(r, t)=E_{0} \frac{\sin (\beta / 2)}{\beta / 2} \sin (k r-\omega t)
$$

with $\beta=k D \sin \theta$. You may use any or none of the following relations in obtaining your answer.

$$
\begin{aligned}
\cos a \cos b & =\frac{1}{2}[\cos (a-b)+\cos (a+b)] \\
\sin a \sin b & =\frac{1}{2}[\cos (a-b)-\cos (a+b)] \\
\sin a \cos b & =\frac{1}{2}[\sin (a+b)+\sin (a-b)] \\
\cos (a \mp b) & =\cos a \cos b \pm \sin a \sin b \\
\sin (a \pm b) & =\sin a \cos b \pm \cos a \sin b
\end{aligned}
$$

$$
\begin{aligned}
&\langle f(t)\rangle=\frac{1}{T} \int_{0}^{T} f(t) d t, \quad T=\text { period } \\
&\left\langle\cos ^{2} \omega t\right\rangle=\frac{1}{2} \\
&\left\langle\cos ^{2} \omega t\right\rangle=\frac{1}{2} \\
&\left\langle\sin ^{2} \omega t\right\rangle=\frac{1}{2} \\
&\langle\sin \omega t\rangle=0 \\
&\langle\cos \omega t\rangle=0
\end{aligned}
$$

