Physics 7C Second Midterm: Lecture 2
Solutions

1) The first step is to see where the first diffraction minimum lies, and then to use that location to find how many orders (may be a fraction) of interference occur.

$$\text{slit width } a = 6 \times 10^{-5} m, \text{ slit separation } d = 6 \times 10^{-4} m, \lambda = 6.00 \times 10^{-9} m$$

The pattern for diffraction is given by

$$I(\theta) = I(0)\frac{\sin^2(\pi a \sin\theta/\lambda)}{(\pi a \sin\theta/\lambda)^2}$$

which has its first minimum when $\sin^2(\pi a \sin\theta/\lambda) = 0$. This occurs when

$$\sin\theta = \lambda/a$$

If this coincides with an interference maximum $m\lambda = d\sin\theta$ locates the mth order:

$$m = \frac{d\sin\theta}{\lambda} = \frac{d}{\lambda}\frac{\lambda}{a} = \frac{d}{a} = 10$$

This would be the tenth order interference maxima, but it is not visible, as the coincidence of the diffraction minimum causes the intensity to be zero there. Since the central interference maximum is

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In[15]:= Plot[(Sin[Pi*100*Sin[θ]]/(Pi*100*Sin[θ]) * Cos[Pi*1000*Sin[θ]])^2, {θ, -0.01, 0.01}]
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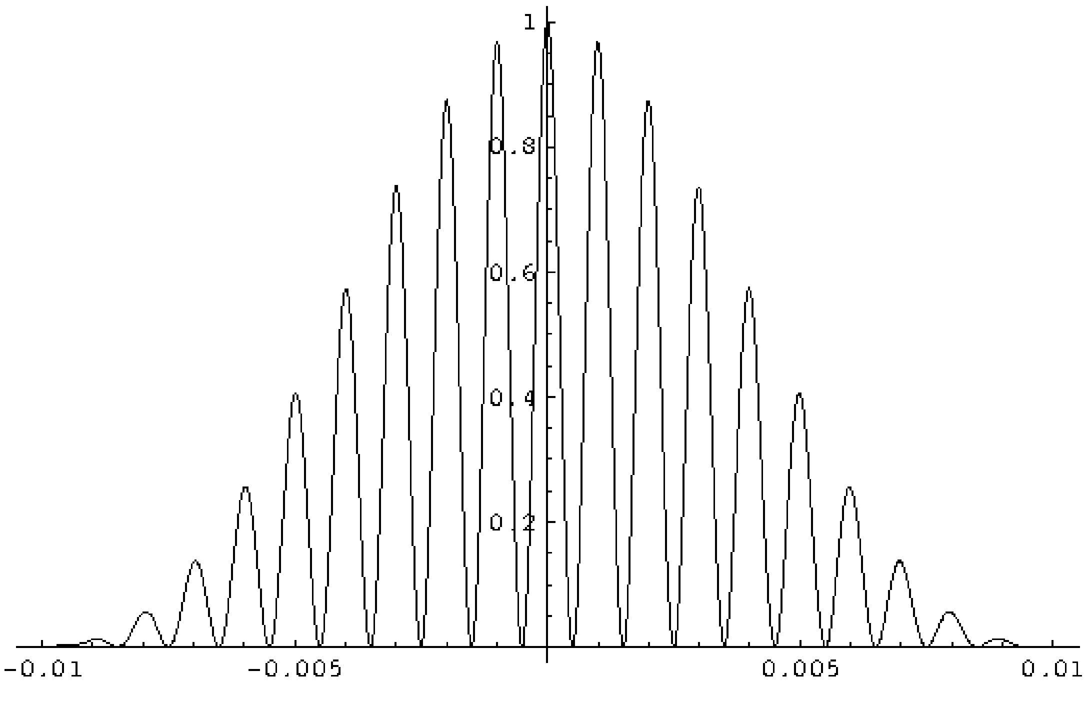
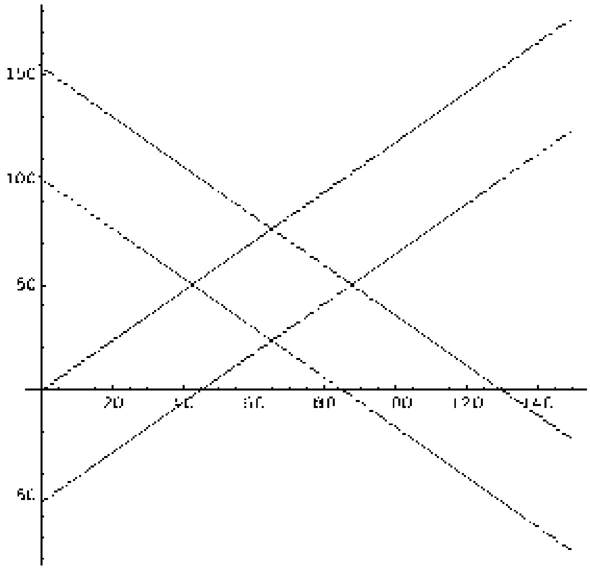


2) Let S be the Earth's reference frame, and S' be the reference frame of the ship moving to the right at speec



a)

b) Measurement of the spaceship from the Earth's frame must use $\delta t = 0$. The length of the ship as measured in its own rest frame (where the time interval is immaterial) is $\Delta x' = 100m$. Use the Lorentz tranformation from S to S':

$$\begin{aligned} \Delta x' &= \gamma(\Delta x - \beta c \Delta t) = \gamma(\Delta x) \\ \Delta x &= \frac{1}{\gamma}\Delta x' = \sqrt{1-\beta^2}\Delta x' \\ \Delta x &= \sqrt{1-.85^2}100m = 52.7m \end{aligned}$$

another picture

c) The speed u of the two ships are known in the S frame. The question is the magnitude of the velocity u' of the left-moving ship in the S' frame. Using the velocity addition formula:

$$\frac{u'}{c} = \frac{(u-v)/c}{1-uv/c^2} = \frac{-.85-.85}{1-(-.85)(.85)} = -.987$$

$$|u'| = .987c$$

d) Procede similarly to part b, this time using S'' being the rest frame of the left-moving ship. Again, measurement of the spaceship must use $\delta t' = 0$ in the S' frame. The length of the ship as measured in its own rest frame (where the time interval is immaterial) is $\Delta x'' = 100m$. Use the Lorentz tranformation from S' to S'', with the speed between them .987 c as found in part (c).

$$\begin{aligned} \Delta x'' &= \gamma(\Delta x' - \beta' c \Delta t') = \gamma(\Delta x') \\ \Delta x' &= \frac{1}{\gamma}\Delta x'' = \sqrt{1-\beta'^2}\Delta x'' \\ \Delta x' &= \sqrt{1-.987^2}100m = 16.1m \end{aligned}$$

e) From part (a), we see that in the Earth's frame, $\Delta x = 0$. As always, $\Delta x' = 100m$ and $\Delta t'$ is unknown.

$$\Delta x' = \gamma(\Delta x - \beta c \Delta t) = -\gamma(\beta c \Delta t)$$

$$\Delta t = \frac{-1}{\gamma \beta c}\Delta x' = \frac{-100m}{-.85c}\sqrt{1-.85^2} = 2.07 \times 10^{-7}s$$

f) This time, solve for $\Delta t'$ with $\Delta t', \Delta x, \Delta x'$ as before.

3) a) In the original spaceship rest frame, the momentum is $p' = 0$, and the energy is $E' = Mc^2$. After ejecting the fuel (at rate $v = -c/2$ with respect to the V_0 frame)

$$\Sigma p'_f = \frac{M' v'_f}{\sqrt{1-(v'_f/c)^2}} - \frac{mc/2}{\sqrt{1-.5^2}} = 0$$

by momentum conservation. Energy is also conserved; the final energy can be written as:

$$\Sigma E'_f = \frac{M' c^2}{\sqrt{1-(v'_f/c)^2}} + \frac{mc^2}{\sqrt{1-.5^2}}$$

For an individual particle, can express $\beta E = cp$. Use $E'_f = \frac{p'_f c^2}{v'_f}$ to substitute the spaceship's momentum:

$$\begin{aligned}
\Sigma E'_f &= \frac{c^2}{v'_f}\frac{mc/2}{\sqrt{1-.5^2}} + \frac{mc^2}{\sqrt{1-.5^2}} = Mc^2 \\
\frac{c^2}{v'_f} &= \frac{\sqrt{3}}{mc}\left(Mc - \frac{2mc}{\sqrt{3}}\right) \\
\frac{c}{v'_f} &= \sqrt{3}\frac{M}{m} - 2 \\
v'_f &= \frac{c}{\sqrt{3}\frac{M}{m}-2}
\end{aligned}$$

b) The final speed of the spaceship in the frame moving at speed V_0 with respect to the Earth is $v'_f = \frac{c}{\sqrt{3}\frac{M}{m}-2}$. Use the standard velocity boost to find the speed in the Earth's frame (or boost backwards by $-V_0$).

$$\begin{aligned}
v_f &= \frac{v'_f - (-V_0)}{1 - v'_f(-V_0)/c^2} \\
&= \frac{\frac{c}{\sqrt{3}\frac{M}{m}-2} + V_0}{1 + \frac{V_0}{c}\frac{1}{\sqrt{3}\frac{M}{m}-2}} \\
&= \frac{c + V_0(\sqrt{3}\frac{M}{m} - 2)}{\sqrt{3}\frac{M}{m} - 2 + \frac{V_0}{c}}
\end{aligned}$$

4) The photoelectirc effect demonstrates the linear relationship between energy and frequency:

$$E_\nu = h\nu$$

If an electron absorbs a photon, then it acquires the full $h\nu$, but that result must be greater than the potential binding the electron to the atom, so that the kinetic energy of the electron is

$$E_k = h\nu - \phi$$

This much kinetic energy is stopped by a potential

$$eV_{stop} = h\nu - \phi$$

Using units of electronVolts we have

$$\begin{aligned} 1eV &= h\frac{c}{430nm} - \phi \\ 0.4eV &= h\frac{c}{550nm} - \phi \end{aligned}$$

Subtracting the second equation from the first:

$$\begin{aligned} 0.6eV = & \quad hc\left(\frac{1}{430nm} - \frac{1}{550nm}\right) \\ h = & \quad \frac{0.6eV}{3\times 10^8 m/s} \cdot 1970nm = 3.94 \times 10^{-15} eV \cdot s \end{aligned}$$