MATH 54 MIDTERM 1 – February 15 2017 5:10-6:30pm

Your Name Student ID	

Please exchange student IDs to record the

names of your two closest seat neighbors	

Do not turn this page until you are instructed to do so.

No material other than simple writing utensils may be used. Show all your work in this exam booklet. There are blank pages for scratch work. **Do not remove any pages!**
If you want something on an extra page to be graded, label it by the problem number and write "XTRA" on the page of the actual problem.
In the event of an emergency or fire alarm leave your exam on your seat and meet with your GSI or professor outside.

This exam is designed to be independent of your B-value. It consists of 7 problems. Point values are indicated in brackets to the left of each problem and are roughly indicate the maximal number of minutes that you should spend on them. Partial credit is given for explanations and documentation of your approach, even when you don't complete the calculation. In particular, if you recognize your result to be wrong (e.g. by checking!), stating this will yield extra credit.

When asked to explain/show/prove, you should make clear and unambiguous statements, using a combination of formulas and words or arrows. Graders will disregard formulas whose meaning is unclear.

You may use any theorems or facts stated in lecture or the book sections covered by the course up to Feb.15 – after stating them clearly. If you wish to use theorems or facts that you may know from other sources, you need to include proofs that derive them from the current course material.

[4] **1a)** *Rewrite the system of linear equations*

$$\begin{aligned} 2x_2 + 2x_4 &= 8 \\ x_1 - x_3 &= 0 \\ x_1 + x_2 - x_3 + x_4 &= 4 \end{aligned}$$

in terms of a linear transformation $T : \mathbb{R}^n \to \mathbb{R}^m$*, a vector* $\mathbf{b}$ *in* $\mathbb{R}^p$*, and an unknown* $\mathbf{x}$ *in* $\mathbb{R}^q$*. In particular, specify* T *and* $\mathbf{b}$ *explicitly. (This involves a choice of appropriate integers* m, n, p, q*.)*

[7] **1b)** *Find the solution set of the system in 1a) and write it in parametric vector form.*

[6] **1c)** *Find the solution set for the homogeneous system associated to the inhomogeneous system in 1a).*
(You can use any method but should show your work or explain your reasoning.)

[3] **1d)** *State (but do not prove) a relationship between the solution sets in 1b) and 1c).*

[10] **2)** *Let* $T : V \to W$ *be a linear transformation between vector spaces* V, W.
Give the definition of $\mathrm{kernel}(T)$ *and state what it is a subspace of.*
Then prove your statement without appealing to theorems from book or lecture.

[10] **3)** *Find the general solution of* $T(\mathbf{x}) = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$ *and explain why there cannot be any other solutions, using only definitions and the following information (no theorems etc.):*

1.) $T : \mathbb{R}^3 \to \mathbb{R}^2$ *is a linear transformation,*

2.) $\operatorname{kernel} T = \operatorname{span}\left\{ \begin{bmatrix} 3 \\ 0 \\ -2 \end{bmatrix} \right\}$,

3.) $T\left(\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$.

[3] **4a)** *Consider the functions* t^2*,* $1+t$*, and* $(1+t)^2$ *as vectors in* $\mathbb{P}_2$*, the vector space of polynomials up to degree 2. State the definition of what it would mean for these three vectors to span* $\mathbb{P}_2$*.*

[7] **4b)** *Using your definition from 4a), show that* t^2*,* $1+t$*, and* $(1+t)^2$ *do span* $\mathbb{P}_2$*.*

[6] **5a)** *Determine the standard matrix associated to the linear transformation*

$$T : \mathbb{R}^4 \to \mathbb{R}^4, \qquad \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} \mapsto \begin{bmatrix} 3b - 2d + a \\ 4d + 2b - c \\ 5c - 3d \\ 6d \end{bmatrix}.$$

[4] **5b)** *Explain whether T is one-to-one, onto, both, or neither, by relating these properties to uniqueness and existence of solutions to equations $T(\mathbf{x}) = \mathbf{b}$.*

[5] **6a)** *Replace the* $\cdot$ *below with entries* 1, 0, *or* $\star$ *(to denote entries that can be any real number) to create a list of all possible* 2×3 *matrices in reduced echelon form.* *(Hint: There are less than 8 different forms.)*

$$\begin{bmatrix} \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \end{bmatrix} \qquad \begin{bmatrix} \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \end{bmatrix} \qquad \begin{bmatrix} \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \end{bmatrix} \qquad \begin{bmatrix} \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \end{bmatrix}$$

$$\begin{bmatrix} \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \end{bmatrix} \qquad \begin{bmatrix} \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \end{bmatrix} \qquad \begin{bmatrix} \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \end{bmatrix} \qquad \begin{bmatrix} \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \end{bmatrix}$$

[5] **6b)** *State (don't prove) criteria for reading off from the reduced echelon form of a matrix whether the associated linear transformation is one-to-one or onto.* *(Hint: You probably already used these in 5b.)* *Then apply these criteria to label the matrices above with the one-to-one/onto properties that the associated linear transformations have.*

[6] **7a)** *State a pivot criterion for deciding whether the columns of a matrix are linearly independent. Then apply it to decide whether the vectors* $\begin{bmatrix} 5 \\ 3 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 3 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$ *are linearly independent in* $\mathbb{R}^3$.

[4] **7b)** *State the definition of* $\begin{bmatrix} 5 \\ 3 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 3 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$ *being linearly independent, and check this condition.*