

## **Final Exam** 12/15/2016

Name:\_\_\_\_\_

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Version 58099549

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Instructions:

E7

- 1. Write your full name and Student ID on the question sheet
- 2. Write your full name on the front of the bubble sheet
- 3. If your student ID number has 8 digits, start with the left box and bubble then in (leaving the right two boxes blank). If your student ID number has 10 digits, just use all boxes
- 4. Record your answers on the bubble sheet (use pencil). There is one correct answer for each question. Multiple bubbles, incomplete bubbles, or stray marks will be marked incorrect.
- 5. You may use one 8.5" by 11" cheat sheet(both sides) for this exam provided you do not disturb those sitting nearby.
- 6. No electronic devices are permitted in your work area.
- 7. There will be no questions regarding the exam during the examination time except in cases where there is a missing page or printing problem with your exam.
- 8. For convenience, you may separate the bubble sheet from the question sheet, return both after the exam.

1. The exponential function  $e^x$  can be computed using the series:

$$e^{x} = 1 + x + \frac{x^{2}}{2!} + \frac{x^{3}}{3!} + \frac{x^{4}}{4!} + \dots$$

The function exppo below was developed to compute the first N terms of this series, but is incomplete. When inserted into the incomplete line, which of these options will correctly compute this series?

```
function sum = exppo(x,N)
sum=0;
for i=1:N
    sum = % incomplete line
end
end
(a) sum + x^(i)/factorial(i)
(b) x^(i)/factorial(i)
(c) sum + x^(i-1)/factorial(i-1)
(d) x^(i-1)/factorial(i-1)
```

- (e) sum + x<sup>(i+1)</sup>/factorial(i+1)
- 2. A straight line is given to pass through the points (1,5) and (2,8). The equation of the straight line is of the form  $y = a_0 + a_1 x$ . We wish to set up a system of equations to solve for the unknown coefficients  $a_0$  and  $a_1$  (in that order). Which line of code below will accomplish this?
  - (a) [1 1; 1 2]\[5;8]
  - (b) [1 2; 1 1]\[5;8]
  - (c) [1 5; 1 8]\[1;2]
  - (d) [1 8; 1 5]\[1;2]
  - (e) [1;2]\[1 5; 1 8]
- 3. You are using the bisection method to find the root of a function f(x). The current interval [a, b]. You know f(a) = -2, f(b) = 5, and f((a + b)/2) = 2. What is the interval of the next iteration?
  - (a) [a, b]
  - (b)  $[a, \frac{(a+b)}{2}]$
  - (c)  $[\frac{(a+b)}{2}, b]$
  - (d)  $[a, \frac{(a+b)}{4}]$
  - (e)  $\left[\frac{(a+b)}{4}, \frac{(a+b)}{2}\right]$

4. You are attempting to solve for the root of a function f(x) using the Newton-Raphson Method. Assume an initial guess x0, and that you are writing a while loop to solve for the root. You may assume that f is a function handle, and df is a handle to the derivative of f(x). Complete the incomplete line of code by choosing from the options below.

```
tol= 1e-6;
res=10000;
x= x0;
while (abs(res)>tol)
x= ; % incomplete
res= f(x);
end
```

(a) f(x)/df(x)

- (b) x + f(x)/df(x)
- (c) -f(x)/df(x)
- (d) (x f(x))/df(x)
- (e) x f(x)/df(x)
- 5. Given the function myFun(a1, a2, a3, a4):

```
function [x] = myFun(a1, a2, a3, a4)
    if nargin == 3
        a4 = 1;
    else
        a4 = 2;
    end
        x = a1 + a2 + a3 + a4;
end
```

after executing the following code, what is the value of y?

```
y = myFun(0,1,1,1) + myFun(0,1,1)
(a) 5
(b) 7
(c) 8
(d) 9
(e) 10
```

6. We have a set of 7 data points of the form (x, y) where x represents the x coordinate and y represents the y coordinate. We are trying to fit a polynomial of the form

$$f(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3$$

To do this we set up a system of equations in the form Az = b where z represents the unknown coefficients. What is the size of A?

- (a)  $7 \times 4$
- (b)  $4 \times 7$
- (c)  $7 \times 3$
- (d)  $2 \times 4$
- (e)  $7 \times 2$
- 7. Given the function bsearch, which is defined below:

```
function [Idx,Mid] = bsearch(M, Key)
    Left = 1;
    Right = numel(M);
    while Right>Left
        Mid = floor((Left+Right)/2);
        if M(Mid)<Key</pre>
            Left = Mid+1;
        else
             Right = Mid;
        end
    end
    if M(Left) ==Key
        Idx = Left;
    else
        Idx = [];
    end
end
```

After executing the following code, what is the value of Mid?

```
A = [-2, -1, 0, 1, 2, 3, 4, 5, 6, 7];
[K,Mid] = bsearch(A, 5);
(a) 5
(b) 6
(c) 7
(d) 8
(e) 9
```

8. What does the function myMystery1 do?

```
function y = myMystery1(n)
    if n == 0
        y = 1
    else
        y = 10 * myMystery1(n-1)
    end
end
```

- (a) Compute y = 10 \* n
- (b) Compute  $y = 10^n$
- (c) Compute y = factorial(n)
- (d) Compute y = 10 \* (n 1)
- (e) Compute y = factorial(10)
- 9. Given the function fib, which computes Fibonacci numbers in recursive form:

```
function f = fib(n)
    if n==1
        f = 0;
    elseif n == 2
        f = 1;
    else
        f = fib(n-1) + fib(n-2);
    end
end
```

After executing the following code, what is the value of

y = fib(6)/fib(7);
(a) 5/8

- (b) 8/5
- (c) 8/13
- (d) 13/8
- (e) 5/13

10. Given the function myMystery3:

```
function f = myMystery3(list)
   possible1 = list(1)
   possible2 = myMystery3(list(2:end));
   if possible1 > possible2
      f = possible1
   else
      f = possible2
   end
end
```

After executing the following code, what is the value of a?

```
a = myMystery3([5 4 3 9])
```

- (a) 3
- (b) 9
- (c) 5
- (d) Matlab returns an error: Index exceeds matrix dimensions.
- (e) Matlab returns an error: Out of memory. (i.e. infinite recursion)

11. Given a function myMystery2, which has a subfunction add as shown below.

```
function f=myMystery2(n)
  f=add(n,1,2);
end
function a=add(n,x0,x1)
  if(n==1)
    a=x0;
  else
    if(n==2)
        a=x1;
    else
        a=add(n-1,x1,x0+x1);
    end
  end
end
```

After executing the following code, what is the value of y?

```
y = myMystery2(3)
```

- (a) 1
- (b) 2
- (c) 3
- (d) 4
- (e) 5

12. The Taylor expansion of sin(x) is given by:

$$\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} - \dots$$
(1)

An incomplete coding for this series including n terms is given below:

To complete the function, incomplete lines 1 and 2 need to be replaced by

(a) while i < n result = result + (-1)^(n-1)\*x^(2\*n-1)/fact(2\*n-1);
(b) while i < n result = (-1)^i\*x^(2\*i+1)/fact(2\*i+1);
(c) while i < n result = result + (-1)^(i-1)\*x^(2\*i-1)/fact(2\*i-1);
(d) for i = 1:n result = result + (-1)^i\*x^(2\*i+1)/fact(2\*i+1);
(e) for i = 1:n

result = result +  $(-1)^{(i-1)} \times^{(2 \times i - 1)} / fact(2 \times i - 1);$ 

13. After executing the following code,

```
tspan = linspace(0,10,101)';
odefun = @(t,x) t;
[a,b] = ode45(odefun,tspan,0);
[c,d] = size(a);
```

Which of the following is the best choice?

- (a) c = 101
- (b) b(1) is 0
- (c) a is same as tspan
- (d) Both (a) and (b) are correct
- (e) Above are all correct
- 14. A student wanted to solve the ODE

$$y' = t$$

at t = 2 with initial condition y(1) = 0. He chose Forward Euler and came up with the following code. Now he feels something is wrong and needs your help with debugging.

```
dt = 0.01;
                              % Line 1
y0 = 0;
                              % Line 2
t = 0;
                              % Line 3
y = y0;
                              % Line 4
yprimeh = @(t,y) t;
                              % Line 5
for i = 1:100
                              % Line 6
    yp = yprimeh(t(end));
    yn = y(end) + yp*dt;
    t = [t, tn];
    y = [y, yn];
    tn = t(end) + dt;
end
```

Which of the following change will fix the problem?

(a) Line 1: dt = 0.02
(b) Line 2: y0 = 1
(c) Line 3: t = 1
(d) Line 6: for i = 1:200
(e) Line 6: for i = 1:50

15. Given the ODE

y'' + 3y' + y = 0

Assume  $\mathbf{x} = [\mathbf{y}'; \mathbf{y}]$ 

Reduce above second order ODE into two first order ODEs and obtain

 $\mathbf{x}' = \mathbf{A}\mathbf{x}$ 

Which of the following Matlab expressions would return a value of logical(0) (false)?

(a) A(1,1) == -3
(b) A(1,2) == -1
(c) A(2,1) == 1
(d) A(2,2) == -1
(e) sum(sum(A)) == -3

16. After executing the following code:

```
x = linspace(0,100,101);
dx = diff(x);
y = linspace(0,200,101);
dy = diff(y);
dydx = dy./dx;
n = length(dydx);
plot(x,dxdy)
```

Which of the following is **not true**?

- (a) n is equal to 100
- (b) dydx is a scalar
- (c) dydx stores the slopes of y = 2x
- (d) The above code will return an error
- (e) dx(3) is equal to 1

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  - 17. After executing the following code, which of the following is true?

```
a = [3,1,5,2,1];
da = polyder(a);
a1 = polyint(da);
ia1 = polyint(a1);
a2 = polyder(ia1);
```

- (a) a1 is the same as a
- (b) a2 is the same as a1
- (c) all represents the polynomial  $y = 3x^4 + x^3 + 5x^2 + 2x + 1$
- (d) a represents the polynomial  $y = x^4 + 2x^3 + 5x^2 + x + 3$
- (e) a2(end) is equal to 1;

Note: polyder and polyint are two Matlab built-in functions.

```
>> help polyder
 polyder Differentiate polynomial.
    polyder(P) returns the derivative of the polynomial whose
    coefficients are the elements of vector P.
    polyder(A,B) returns the derivative of polynomial A*B.
    [Q,D] = polyder(B,A) returns the derivative of the
    polynomial ratio B/A, represented as Q/D.
    Class support for inputs u, v:
       float: double, single
>> polyder([2,1,1])
ans =
     4
           1
>> help polyint
 polyint Integrate polynomial analytically.
    polyint(P,K) returns a polynomial representing the integral
    of polynomial P, using a scalar constant of integration K.
    polyint(P) assumes a constant of integration K=0.
    Class support for inputs p, k:
       float: double, single
```

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18. After executing the following code, which of the following is **not true**?

```
I1 = trapz([1,9]);
I2 = trapz([2,3],[1,9]);
I3 = trapz([2,2.5],[1,9]);
I4 = trapz([1,1.5,2.5],[2,4,6]);
I5 = trapz([2,3,4]);
```

- (a) I1 is the same as I2
- (b) **I2** is 10
- (c) **I3** is 2.5
- (d) I4 is 6.5
- (e) **I5** is 6

Note: trapz is a Matlab built-in function.

```
>> help trapz
 trapz Trapezoidal numerical integration.
    Z = trapz(Y) computes an approximation of the integral of Y via
    the trapezoidal method (with unit spacing). To compute the integral
    for spacing different from one, multiply Z by the spacing increment.
    For vectors, trapz(Y) is the integral of Y. For matrices, trapz(Y)
    is a row vector with the integral over each column. For N-D
    arrays, trapz(Y) works across the first non-singleton dimension.
    Z = trapz(X, Y) computes the integral of Y with respect to X using
    the trapezoidal method. X and Y must be vectors of the same
    length, or X must be a column vector and Y an array whose first
    non-singleton dimension is length(X). trapz operates along this
    dimension.
    Z = trapz(X,Y,DIM) or trapz(Y,DIM) integrates across dimension DIM
    of Y. The length of X must be the same as size(Y,DIM)).
Example:
>> trapz([1,2])
ans =
   1.5000000000000000
>> trapz([1,2,3])
ans =
     4
```

19. After executing the following code, what is the value of a?

```
fh = @(x) 0*x+1;
I11 = integral(fh,0,0);
I12 = integral(fh,0,1);
a = [I11 I12];
```

- (a) [0 0]
- (b) [1 0]
- (c) [0 1]
- (d) [1 1]
- (e) **[1 2]**

Note: integral is a Matlab built-in function.

```
>> help integral
integral Numerically evaluate integral.
Q = integral(FUN,A,B) approximates the integral of function FUN from A
to B using global adaptive quadrature and default error tolerances.
FUN must be a function handle. A and B can be -Inf or Inf. If both are
finite, they can be complex. If at least one is complex, integral
approximates the path integral from A to B over a straight line path.
.....
```

20. Given the figure on page 14,

What is the **forward difference** approximation of  $\frac{df}{dx}$  at x = 2? Use an interval width of  $\Delta x = 1$ .

- (a) -2
- (b) -1.5
- (c) -0.5
- (d) 1
- (e) 2



21. Given the vector,

 $p = [1 \ 2 \ 3 \ 4 \ 5];$ 

which of the following commands would assign to q the value  $\left[5\ 4\ 3\right]?$ 

(a) q = p(5:3)
(b) q = q(end:end-2)
(c) q = p(end:-1:3)
(d) q = p(end:end-2)
(e) q = q(5:3)

22. Given the vectors,

x = [4 8 9]; y = [2 2 3];

which of the following commands would compute and assign to z the value [2 4 3]?

- (a)  $z = x \setminus y$
- (b) z = x/y
- (c) z = x.y
- $(d) \ z = x. \backslash y$
- (e) z = x./y
- 23. Which of the following commands, when filling in the incomplete line, would assign to a the value of [-2 -3 -1 -2]?

a =  $[1 -2 -3 \ 0 \ -1 \ -2];$ a = % incomplete line (a) a = a(a<0) (b) a = a(2 3 5 6) (c) a = a<0 (d) a = a([0 1 0 0 1 1]) (e) a = a(negative)

24. After executing the following code, what is the value of y?

p = [1, 1, 3]; x = [1; 2; 0]; y = polyval(p, x) (a) [5; 9; 3] (b) [5; 3; 3] (c) [3; 5; 1] (d) [15; 9; 5] (e) Matlab returns an error 25. After executing the following code, what is the value of G?

```
x = \begin{bmatrix} 0 & 4 & 3 & -1; & 2 & 1 & 1 & -1 \end{bmatrix};

v = x \cdot 2 - 10;

F = sum(v);

G = F(1)

(a) -20

(b) -16

(c) -10

(d) 16

(e) 20
```

26. After executing the following code, what is the value of n?

```
p = 3;
q = 10;
for i = 1:0.5:p
    for j = q:-2:1
        n=i+j;
    end
end
(a) 2.5
(b) 4
(c) 5
(d) 7.5
(e) 10
```

27. After executing the following code, what is the value of n?

```
m=0;
n=0;
while n >= m
    n = n+1;
end
(a) 0
(b) 1
(c) 64
(d) 128
(e) The loop W
```

(e) The loop will run until the user terminates it, so it is impossible to know

28. After executing the following code, what is the value of T?

```
K = 5;
I = -2;
T=-10;
while (I <= K)
I = I + 2;
K=K-I;
T = T+K*I;
end
(a) 0
(b) 8
(c) -4
(d) -8
(e) -14
```

29. After executing the following code, what is the value of f?

```
A = [4 2 16 12 4 3];
B = [12 3 1 10 -1 7];
f = find(A<B)
(a) [4 2 3]
(b) [1 2 6]
(c) [12 3 7]
(d) [4 12 2 3 3 7]
(e) 3</pre>
```

30. After executing the following code, what is the value of B?

```
A = ones(4, 4);
for c = 1:4
    for r = 1:4
        if r == c
             A(r,c) = 2;
        else
             A(r,c) = 1;
        end
    end
end
B = A(1, :)
 (a) [1 2 1 1]
(b) [2 1 1 1]
 (c) [1 2 1 1]
(d) [2 2 1 1]
 (e) [2 2 1 2]
```

31. Which of the following commands would plot discrete points of X vs. Y with **crosses** in blue color?

(a) plot(X, Y, 'b', '.')
(b) plot(X, Y, 'b', 'y')
(c) plot(X, Y, 'b')
(d) plot(X, Y, 'b+')
(e) plot(X, Y, 'r+')

32. After executing the following code, what is the value of R?

```
clear all
x = [1 -2 2 1];
y = [5 -1 4 1];
R = [y(x == max(x)), y(max(x))]
(a) [4, -1]
(b) [4, 4]
(c) [1, 4]
(d) [5, -1]
(e) Matlab Error: Undefined function or variable 'R'.
```

33. Given the function f,

function x = f(y)x = y + 3; end

After executing the following code, what is the value of y?

```
y = f(fzero(@f, 1))
(a) -2
(b) -1
(c) 0
(d) 3
(e) 6
```

34. After executing the following code, what is y?

```
y = class('2')
```

- (a) integer
- (b) double
- (c) char
- (d) cell
- (e) E7

35. Given the function myRecursion, defined as:

```
function x = myRecursion(y)
    if y == 1
        x = 1;
    elseif y == 2
        x = 2;
    else
        x = myRecursion(y+1) + myRecursion(y);
    end
end
```

what is the output of the following command?

myRecursion(0)

- (a) 4
- (b) 3
- (c) 2
- (d) 1
- (e) Matlab returns an error

36. After executing the following code, what is the value of b?

a = [1, 3, 7, -1, 3, -1, -1]; b = [mean(a), median(a), mode(a)] (a) [1.5714, 1.5714, 1] (b) [1.5714, 1.5714, 3] (c) [1.5714, -1, 2] (d) [1.5714, 1, -1] (e) [1.5714, 1, 3] 37. After executing the following code,

```
A = [1 2 3; 2 4 6; 3 6 9];
B = [A; A];
C = [A; A(1, :)];
D = [A; zeros(1, 3)];
E = [A; ones(1, 3)];
ra = rank(A);
rb = rank(B);
rc = rank(C);
rd = rank(D);
re = rank(E);
```

which of the following variables has the greatest numerical value?

- (a) ra
- (b) rb
- (c) rc
- (d) rd
- (e) re

38. After executing the code below, what is the value of y?

```
A = \begin{bmatrix} 1 & 2 & 3; \\ 2 & 0 & 1; \\ 3 & 0 & 0 \end{bmatrix};

b = \begin{bmatrix} 10; & 7; & 9 \end{bmatrix};

x = A \setminus b;

y = x (1, 1)

(a) 1

(b) 2

(c) 3

(d) 9

(e) 10
```

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  - 39. Upon executing the following code, which line will cause an error?

```
a = [1 2 3; 4 5 6];
b = [1 2; 3 4; 5 6];
c = a*b;
d = b*a;
f1 = c^2 %LINE 1
f2 = d*b*a %LINE 2
f3 = ((d.^2).*d)^2 %LINE 3
f4 = c*a*d*b %LINE 4
f5 = 3*a^2 %LINE 5
```

- (b) Line 2
- (c) Line 3
- (d) Line 4
- (e) Line 5
- 40. After executing the following code, you would expect the value of B to be closest to which value?

N = 10000; A = 5\*randn(N,1) + 2\*rand(N,1); B = mean(A) (a) 0
(b) 1
(c) 2
(d) 5
(e) 6 41. After executing the following code, you would expect which variable to have the largest numerical value?

```
r1 = 2 + 3*randn(50000,1);
r2 = 1 + 5*randn(10000,1);
a = mean(r1);
b = mean(r2);
c = std(r1);
d = std(r2);
e = a+b;
(a) a
(b) b
(c) c
(d) d
```

(e) e

42. After executing the following code, what is the value of a?

```
X= [7; 3; 2; 4];
a= [mean(X), median(X)]
(a) [3.5, 3.5]
(b) [4.0, 3.5]
(c) [5.33, 3.5]
(d) [4.0, 4.0]
(e) [NaN, NaN]
```

43. The  $1 \times 100$  row vector **X** has already been loaded into the MATLAB workspace and contains the numerical results of 100 repetitions of an experiment (all values of **X** are greater than 1). After the following code is executed, which of the following statements is NOT true?

```
m1 = mean(X);
m2 = mean(5+X);
m3 = mean(3*X);
s1 = std(X);
s2 = std(5+X);
s3 = std(3*X);
```

- (a) m2 is greater than m1
- (b) m3 is greater than m1
- (c) s2 is greater than s1
- (d) s3 is greater than s1
- (e) m1 is greater than 1
- 44. We have an cell array C and we wish to append the number 1 to it. Which of the following commands will NOT achieve this goal?
  - (a) C = [C {1}] (b) C{end+1} = 1
  - (c) C(end+1) = {1}
  - (d)  $C = \{ C\{:\} 1\}$
  - (e) C(end+1) = 1

45. How do you use error function **erf** or complementary error function **erfc** in Matlab to calculate the probability of x > -1 if the random variable x follow standard normal distribution? It is known that the probability density function follows

$$f(x) = \frac{1}{\sqrt{2\pi}} \exp(-\frac{x^2}{2})$$

An error function is defined as

$$\operatorname{erf}(b) = \frac{2}{\sqrt{\pi}} \int_0^b \exp(-x^2) dx$$

and complementary error function is

$$\operatorname{erfc}(b) = \frac{2}{\sqrt{\pi}} \int_{b}^{\infty} \exp(-x^2) dx$$

- (a) 1-erf(-1)
- (b)  $\operatorname{erf}(1/\sqrt{2})/2 + 1/2$
- (c) erfc(-1)/2
- (d)  $erfc(-1/\sqrt{2})$
- (e)  $\operatorname{erfc}(-1/2)/2$

46. Which of the following is NOT correct to access the value 20 in the cell array data?

data={'name',['double','char'],[4 8 20;3 5 10]}

- (a) data{1,3}(1,3)
- (b)  $data{3}(1,3)$
- (c) data{1,3}(3)
- (d)  $data{3}(5)$
- (e) data{end}(end-1)

47. After the following is executed, what is the value of n ?

```
color='red';
n=0;
switch color
    case {'red', 'black', 'blue'}
        n=n+2;
        color='green';
    case {'yellow', 'green', 'white'}
        n=n+10;
    otherwise
        n=2*n;
end
 (a) 2
(b) 12
 (c) 4
(d) 24
 (e) 10
```

48. After executing the following code, what is the value of y?

```
S=struct('Name', {'Robert', 'Rachael'}, 'Math', {93,95}, 'Chemistry', {78,82});
y = S(2).Name(3)
```

- (a) Rachael
- (b) b
- (c) c
- (d) 95
- (e) Chemistry

49. Structure S is defined as:

```
S=struct('Name',{'Robert', 'Rachael'}, 'Math', {93, 95}, 'Chemistry', {78, 82});
```

We wish to compute the average of the Math scores of the two students and assign it to the variable y. Which of the following Matlab expressions will NOT accomplish this?

```
(a) y = mean(S.Math)
(b) y = 0
for i=1:length(S)
    y = y+S(i).Math/length(S)
end
(c) y = mean([S.Math])
(d) y=0
for i=1:size(S,2)
    y = y+S(i).Math/size(S,2)
end
(e) y = (S(1).Math + S(2).Math)/2
```

50. After executing the following code, how many curves show up in figure 2?

```
figure(1)
hold on;
for i=1:10
    x=linspace(0,1,20);
    p=rand(1,5);
    y=polyval(p,x);
    plot(x,y);
    figure(2)
    hold on;
    y=polyval(-p,x);
    plot(x,y);
end
 (a) 1
(b) 9
 (c) 10
(d) 19
 (e) 20
```