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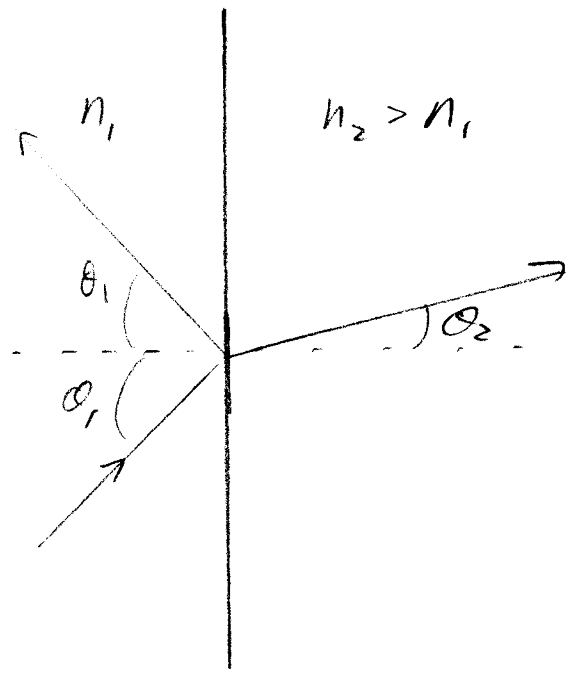
BATTAGLIA MT2 SOLS SPRING 2004

1) $R_{\parallel} = \dfrac{\tan(\theta_1 - \theta_2)}{\tan(\theta_1 + \theta_2)}$



$$R_{\parallel} = 0 \Rightarrow \begin{cases} \theta_1 - \theta_2 = 0 & \text{(TRIVIAL CASE)} \\ \text{OR} & \\ \theta_1 + \theta_2 = \pi/2 & \end{cases}$$

COMBINING 2nd CONDITION & SNELL'S LAW:

$$n_1 \sin\theta_B = n_2 \sin\theta_2 = n_2 \sin(\pi/2 - \theta_B)$$

$$= n_2 \left[\underbrace{\sin \pi/2}_{1} \cos\theta_B - \underbrace{\cos \pi/2}_{0} \sin\theta_B \right]$$

$$= n_2 \cos\theta_B$$

$$\Rightarrow \boxed{\frac{n_2}{n_1} = \tan\theta_B}$$

2) $\theta_B = \tan^{-1}\left(\dfrac{n_2}{n_1}\right) = \tan^{-1}\left(\dfrac{1.33}{1}\right)$

$\boxed{\theta_B = 0.926 = 53.1°}$ FOR WATER

Phys 7C - Spring 2004 - Sect 2 (Battaglia) - M2 Solutions

2: $m_B = 5.2 \text{ GeV}/c^2$ $p_+ = 400 \text{ GeV}/c$

(5pts) 1) $E = \sqrt{m^2c^4 + p^2c^2} = \gamma mc^2 = \sqrt{(mc^2)^2 + (pc)^2}$

note $pc = 400 \text{ GeV} >> mc^2 = 5.2 \text{ GeV}$
ultra relativistic limit. (I'll do full sol'n after)

$E = \sqrt{p^2c^2\left(1 + \left(\frac{mc^2}{pc}\right)^2\right)} = pc\sqrt{1 + \left(\frac{mc^2}{pc}\right)^2} \simeq pc$
for $mc^2 << pc$

$\Rightarrow E = 400 \text{ GeV}$

long way: $E = \sqrt{(pc)^2 + (mc^2)^2} = \sqrt{(400 \text{ GeV})^2 + (5.2 \text{ GeV})^2}$
$= \sqrt{1.6\times10^5 \text{ GeV} + 27.04 \text{ GeV}^2} = 400.034$

so the approx $E \simeq pc$ is valid to
$\frac{400 - 400.034}{400.034} \times 100 = -0.008\%$

Anyway $E = 400 \text{ GeV} = \gamma mc^2 = \gamma\, \underline{5.2 \text{ GeV}}$
$\gamma = 400 \text{ GeV} / 5.2 \text{ GeV} = \boxed{76.9 = \gamma}$

$\gamma >> 1$ - again, ultra-relativistic

(5 pts) 2) moving clocks run slow

A scientist in the lab frame will see a watch strapped to the B-particle moving slow.

$$t_{lab} = \gamma t_B$$

B-particle decays in 1.6×10^{-12} s in it's frame (of course! it's not going to call up the scientist and ask him when 1.6×10^{-12} s has passed on his watch)

$$t_{lab} = \gamma\, 1.6\times10^{-12}\,\text{s} = (76.9)(1.6\times10^{-12}\,\text{s})$$

$$t_{lab} = 1.22\times10^{-10}\,\text{s}$$

now

$x = ut$. what is u?

<u>ultra relativistic!</u> $u \simeq c$!

If you don't beleive me: $\gamma = 76.9 = \frac{1}{\sqrt{1-\left(\frac{u}{c}\right)^2}}$

$$1-\left(\frac{u}{c}\right)^2 = \left(\frac{1}{76.9}\right)^2 = 1.69\times10^{-4}$$

$$\left(\frac{u}{c}\right) = \sqrt{1-1.69\times10^{-4}} = \sqrt{.99983} = 0.999915$$

so if you did this the long way, you wasted a lot of time

$$x \simeq ct = (3\times10^8\,\text{m/s})(1.23\times10^{-10}\,\text{s}) = 0.0369\,\text{m} = 3.69\,\text{cm}$$

$$\boxed{x = 3.69\ \text{cm}} \qquad (t = 1.22\times10^{-10}\,\text{s})$$

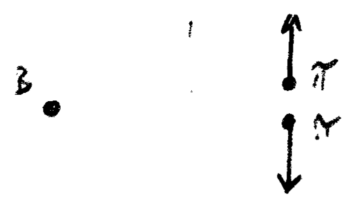
If you did this calculation: $x = ut$, where $t = 1.6\times10^{-12}$ s you got 0 pts (unless you used length contraction). This is core the SR.

3) (15 pts) What is the x-velocity of the π particles in the lab frame?

I'm running in the x-direction along w/ a B-particle moving $\sim c$ w.r.t. the lab frame. π particles are created w/ no x-velocity in that frame (where I'm running). What if I stop?
B-particle will be moving @ $\sim c$
x-velocity of π particles also is c.
my boost is $\simeq c$!
What about y-velocity?

$$u_y = \frac{u_y'}{\gamma\left(1+\frac{v u_x'}{c^2}\right)} = \frac{u_y'}{\gamma\left(1+u_x'/c\right)} \qquad (v \simeq c)$$

note even though $v \simeq c$ I don't plug it in for $\gamma = \frac{1}{\sqrt{1-(v/c)^2}}$
$v \simeq c$ is a better approx than $\gamma \to \infty$
Anyway, we know γ.
What is u_y'?



γ of π-particle in moving frame. (rest frame of B-particle)

$$E_i = m_B c^2 = 5.2\ \text{GeV} \qquad E_f = 2\gamma_\pi' m_\pi c^2 = \gamma_\pi\, 2 \cdot 0.14\ \text{GeV} = 0.28\, \gamma_\pi\ \text{GeV}$$

$$5.2\ \text{GeV} = 0.28\, \gamma_\pi'\ \text{GeV}$$

$$\gamma_\pi = 18.57 \gg 1 = \frac{1}{\sqrt{1-\left(\frac{u_y'}{c}\right)^2}} \Rightarrow u_y' \simeq c\ !$$

really: $1-\left(\frac{u_y'}{c}\right)^2 = 0.0029$

$$\frac{u_y'}{c} = \sqrt{1-0.0029} = 0.9985$$

In B particle frame

Before

B particle. Not moving

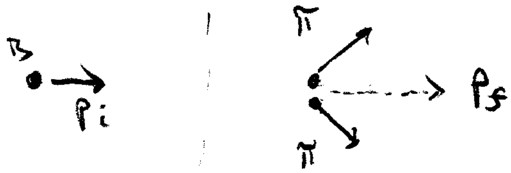
$\boxed{\theta = 180°}$ (+2 pts)

Why does $\theta = 180°$? Conservation of momentum

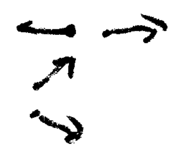
$p_i = 0$ $\qquad$ $p_f = 0$ ← particles must be moving in opposite direction

Why vertical?

When we transform to lab frame, we are going to have to have final momentum



If πs in rest frame of B were horizontal there would be no transform to get to

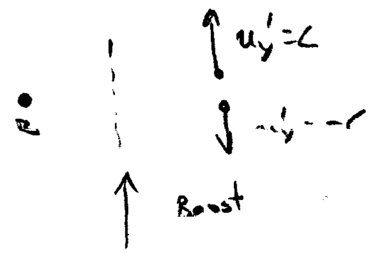


Onward!

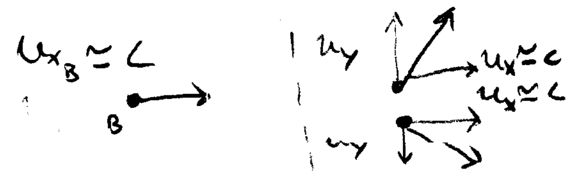
What boost did we need to get to the r.f of the B-particle? The velocity of the B-particle!

$V = c$

so:

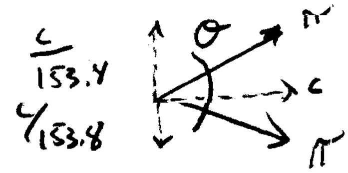


r.f. of B-particle



Lab Frame

$$u_y = \frac{u_y'}{\gamma(1+u_x' v/c^2)} \approx \frac{c}{76.9(1+0)} = \frac{c}{76.9} =$$



Using trig: $\tan\left(\frac{\theta}{2}\right) = \frac{c/76.9}{c} = \frac{1}{76.9} \ll 1$

$$\Rightarrow \tan\frac{\theta}{2} \simeq \frac{\theta}{2} = 1/76.9$$

$$\theta = 2/76.9 = 0.026 \text{ radians} = 1.49^\circ$$

$$\boxed{\theta_{lab} = 0.026\,\text{rad} = 1.49^\circ}$$

-0 if you gave 0.013 rad and sighted $\theta' = \pi/2$ (angle w.r.t. axis as opposed to other particle)

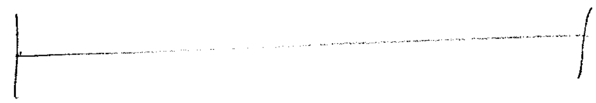
-2 if you gave 0.013 rad for θ & π for θ' or you just didn't tell θ'

Note: Headlight effect: $\frac{\cos(\theta'/2) + v/c}{1 + v/c \cos\theta'/2} = \cos\theta/2$

$\theta'/2 = \pi/2$

$\Rightarrow \cos\theta/2 = v/c$ Now you can't say $v = c$ b/c that gives $\theta = 0$. Indeed, θ is a small #, but it is finite. You can only make approximations if the error due to them is much less than the value you get.

3



S: $x_1 = 0$ $t_1 = 0$ $\qquad$ $x_2 = L$ $t_2 = \text{ANY}$

S': $x_1' = 0$ $t_1' = 0$ $\qquad$ $x_2' = ?$ $t_2' = t_1'$

TRANSFORM:

$$x_2' = \gamma(x_2 - v t_2) \Rightarrow L' = \gamma(L - v t_2)$$

$$t_2' = \gamma\left(t_2 - \frac{v}{c^2} x_2\right) \Rightarrow t_1' = \gamma\left(t_2 - \frac{v}{c^2} L\right) = 0$$

$$\Rightarrow t_2 = \frac{v}{c^2} L$$

COMBINING:

$$L' = \gamma\left(L - \frac{v^2}{c^2} L\right) = L\left(\frac{1 - v^2/c^2}{\sqrt{1 - v^2/c^2}}\right) = \frac{L}{\gamma}$$

$$\Rightarrow \boxed{\Delta L = L(1 - 1/\gamma)}$$

$$\frac{\Delta L}{L} = 1 - \frac{1}{\gamma} = 1 - \sqrt{1 - v^2/c^2} \Rightarrow v = c\sqrt{1 - (1 - \Delta L/L)^2}$$

$$\frac{\Delta L}{L} = 0.01 \Rightarrow \boxed{v = 0.141\,c} \approx \sqrt{2}\,c\left(\frac{\Delta L}{L}\right)$$

Battaglia.

4. 1). Can't.

$P_i = P_f$

$$\Rightarrow \frac{h\nu}{c} = P_1 + P_2 \quad ①$$

$E_i = E_f$

$$\Rightarrow h\nu = \sqrt{P_1^2c^2 + m^2c^4} + \sqrt{P_2^2c^2 + m^2c^4}$$

$$\Rightarrow \frac{h\nu}{c} = \sqrt{P_1^2 + m^2c^2} + \sqrt{P_2^2 + m^2c^2} \quad ②$$

left hand side of ① and ② are the same,

but $\sqrt{P_1^2 + m^2c^2} + \sqrt{P_2^2 + m^2c^2}$ is always

greater than $P_1 + P_2$. So this reaction can't

make both of momentum and energy

2). transform to 0-momentum reference frame.

$E^2 - p^2c^2$ is invariant quantity.

$$(h\nu + mc^2)^2 - (h\nu)^2 = (3mc^2)^2 - 0^2$$

$$\Rightarrow 2h\nu mc^2 + m^2c^4 = 9m^2c^4$$

over.

$$\Rightarrow \quad 2h\nu mc^2 = 8m^2c^4$$

$$\Rightarrow \quad h\nu = 4mc^2$$

(3). $(h\nu + Mc^2)^2 - (h\nu)^2 = (Mc^2 + 2mc^2)^2$ (the same trick as (2))

$$\Rightarrow \quad 2h\nu Mc^2 + M^2c^4 = M^2c^4 + 4mMc^4 + 4m^2c^4$$

$$\Rightarrow \quad 2h\nu Mc^2 = 4Mmc^4 + 4m^2c^4$$

$$\Rightarrow \quad h\nu = 2mc^2 + \frac{2m^2c^2}{M}.$$

because $M \gg m$,

$$h\nu \approx 2mc^2$$

Battaglia

5. a) ~~$\Delta\lambda = \frac{h}{mc}$~~

$$\Delta\lambda = \lambda_2 - \lambda_1 = \frac{h}{mc}(1-\cos\theta)$$

$$\frac{c}{f_2} - \frac{c}{f_1} = \frac{h}{mc}(1-\cos\theta)$$

$$\frac{f_2 - f_1}{f_1 f_2} = -\frac{h}{mc}(1-\cos\theta)$$

$$f_1 = f_2\left(1 + \frac{f_1 h}{mc^2}(1-\cos\theta)\right)$$

$$f_2 = \frac{f_1}{1+\frac{f_1 h}{mc^2}(1-\cos\theta)}$$

$$h(f_1 - f_2) = E_k$$

$$\Rightarrow \quad E_k = hf_1\left(1 - \frac{1}{1+\frac{f_1 h}{mc^2}(1-\cos\theta)}\right)$$

$$= \frac{hf_1}{1+\frac{mc^2}{hf(1-\cos\theta)}}$$

maximum of E_k, when $\theta = \pi$

$$E_k = \frac{hf_1}{1+\frac{mc^2}{2hf}} = 1.06\,MeV$$

2). $hf_1 - E_{k\,max}$

$= hf_1 - \dfrac{hf_1}{1 + \dfrac{mc^2}{2hf_1}}$

when f_1 is very big

$\Rightarrow hf_1 - hf_1\left(1 - \dfrac{mc^2}{2hf_1}\right)$

$= \dfrac{1}{2} m_e c^2$