Key. EE120 MT #2 F2014

Problem 1. Laplace Transform (~~22~~ 29 points)
Answer each part *independently*.

[~~5~~ 7 pts] a. A causal system with input $x(t)$ and output $y(t)$ is described by the differential equation:

$$\frac{d^2}{dt^2}y(t) + 5\frac{d}{dt}y(t) + 9y(t) = \frac{d}{dt}x(t) + 3x(t).$$

$\dot{y}(t) \rightarrow sY(s) - y(0^-)$

$\ddot{y}(t) \rightarrow s^2Y(s) - sy(0^-) - \dot{y}(0^-)$

Find $Y(s)$ for $x(t) = e^{-2t}u(t)$, with $y(0^-) = 1$, $\frac{d}{dt}y(0^-) = 2$.

$s^2Y(s) - sy(0^-) - \dot{y}(0^-) + 5(sY(s) - y(0^-)) + 9Y(s) = sX(s) + 3X(s)$

$Y(s) =$ ____________

$= \frac{s+3}{s^2+5s+9}X(s) + \frac{s+7}{s^2+5s+9}$

$Y(s)[s^2+5s+9] = X(s)(s+3) + \underbrace{sy(0^-)}_{s} + \underbrace{\dot{y}(0^-) + 5y(0^-)}_{7}$

$= \frac{s+3}{(s+2)(s^2+5s+9)} + \frac{s+7}{s^2+5s+9}$

[5 pts] b. A causal system has Laplace Transform:

$$Y(s) = \frac{10(s-1)}{(s+1)(s+10)}. = \frac{A}{s+1} + \frac{B}{s+10}$$

Find $y(t)$.

$y(t) =$ $\left(-\frac{20}{9}e^{-t} + \frac{110}{9}e^{-10t}\right)u(t).$

$(s+1)Y(s)\big|_{s=-1} = A = \frac{-20}{9}$

$(s+10)Y(s)\big|_{s=-10} = B = \frac{10\cdot(-11)}{-9} = \frac{+110}{9}$

[5 pts] c. An LTI causal system with input $x(t)$ and output $y(t)$ has Laplace Transform:

$$H(s) = \frac{Y(s)}{X(s)} = \frac{10(s-1)}{(s+1)(s+10)}. = \frac{10s-10}{s^2+11s+10}$$

Find the linear differential equation which describes the LTI system:

differential equation: ____________

$\ddot{y} + 11\dot{y} + 10y = 10\dot{x} - 10x$

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[~~x~~ pts] d. An LTI system $H(s)$ has the pole-zero diagram shown below, where

$$H(s) = \frac{9(s+1)}{(s+3)^2}.$$

mplete the table for $|H(j\omega)|$, then sketch the magnitude of the frequency response for this system in
e range given.

$$\frac{9\,|j\omega+1|}{|j\omega+3|^2} = \frac{9\cdot\sqrt{1+\omega^2}}{\omega^2+9}$$

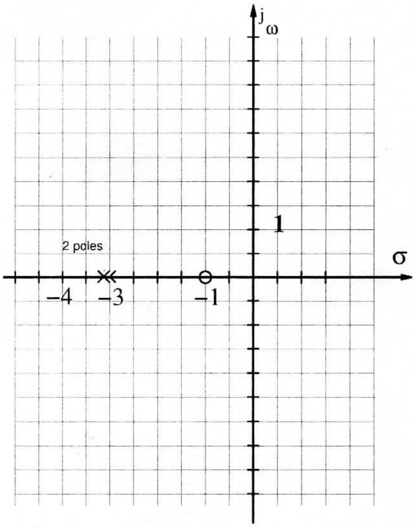
$j\omega$	$\lvert H(j\omega)\rvert$
0j	1
1j	1.3
3j	1.5
10j	0.9
20j	0.45

$$|H(j\cdot 1)| = \frac{9\cdot\sqrt{2}}{10} \approx 1.3$$

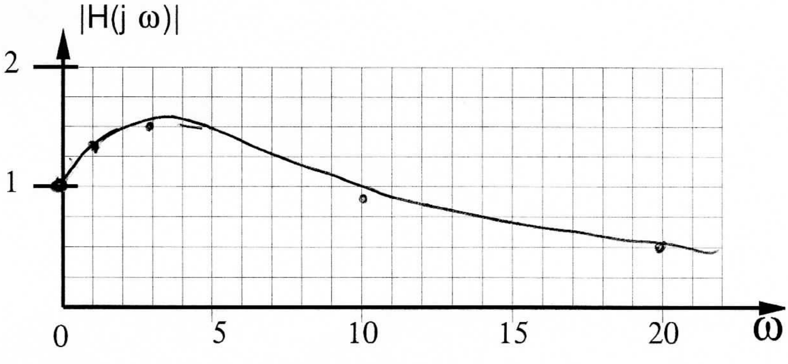
$$|H(3j)| = \frac{9\cdot\sqrt{10}}{2\cdot 9} \approx \frac{\sqrt{10}}{2} \approx 1.5$$

$$|H(10j)| = \frac{9\cdot\sqrt{101}}{109} \approx 0.9$$

$$|H(20j)| = \frac{9\cdot\sqrt{401}}{409} \approx \frac{9}{20} \approx .45$$



Given pole-zero plot.



Sketch $|H(j\omega)|$ here.

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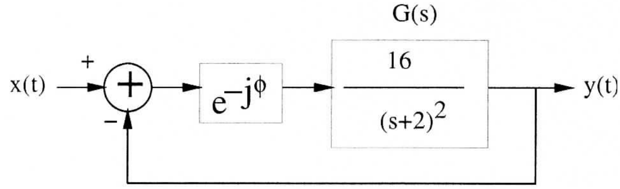
Problem 2. Feedback System (~~26~~ 21 points)



[~~4~~ 5 pts] a. Find the transfer function for the system above which has input $x(t)$ and output $y(t)$, assuming $\phi = 0$.

$$H(s) = \frac{Y(s)}{X(s)} = \frac{16}{(s+2)^2+16}$$

[~~6~~ 7 pts] b. With $\phi = 0$, Find the frequency ω_p at which the gain of $|G(j\omega)|$ is 1.

$$\omega_p = 2\sqrt{3} = \sqrt{12}$$

$$|G(j\omega)| = \frac{16}{|j\omega+2|^2} = \frac{16}{4+\omega^2}$$

$$\omega^2 = 12, \quad = 4 \cdot 3$$

[~~8~~ 9 pts] c. For the frequency ω_p found above, what is the maximum ϕ which could be used before the closed-loop system is unstable (this is the phase margin).

$$\phi < 60^\circ$$

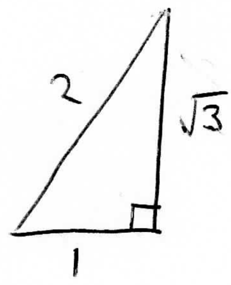
$$\measuredangle G(j\omega_p) = \measuredangle 16 - 2\angle(j\sqrt{12}+2)$$

$$= -2 \angle j\sqrt{3}$$

$$= -120^\circ$$

$$-120^\circ - 60^\circ = -180^\circ$$

$$-\oslash$$



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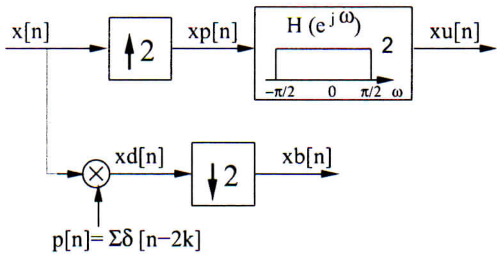
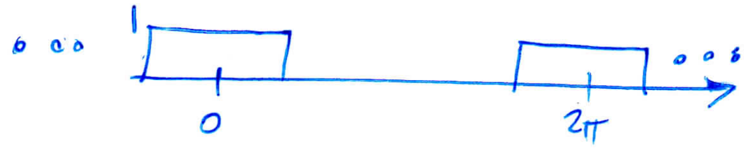
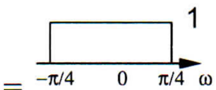
10min

Problem 3. Up and down sampling (24 pts)

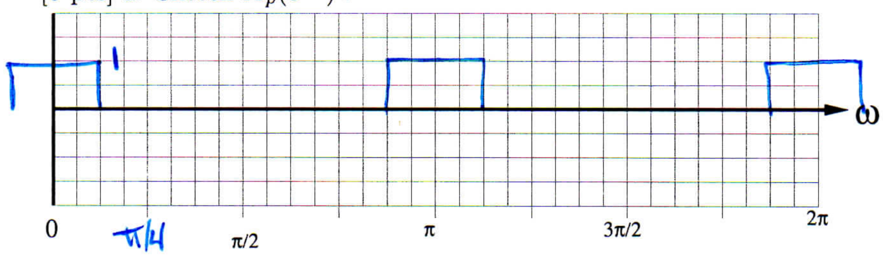
Consider up-sampling a signal $x[n]$ by a factor of 2 and down-sampling $x[n]$ by a factor of 2 as shown in the block diagram below. Sketch the frequency response for the signals shown, indicating key amplitudes and locations. (Note all signals are real and even, thus the spectra are also real and even).

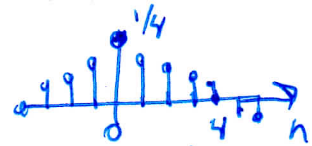




Where $x[n] = \frac{\sin n\pi/4}{\pi n}$, and $X(e^{j\omega}) =$ 

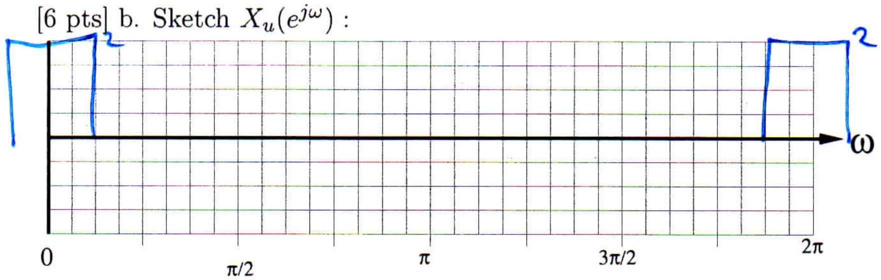
[6 pts] a. Sketch $X_p(e^{j\omega})$:





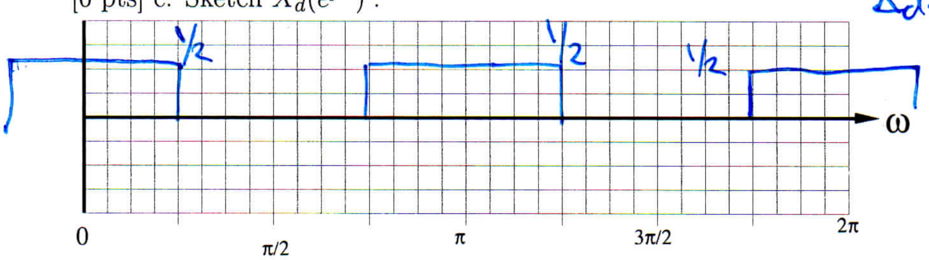
$X_p(e^{j\omega}) = X(e^{2j\omega})$

[6 pts] b. Sketch $X_u(e^{j\omega})$:

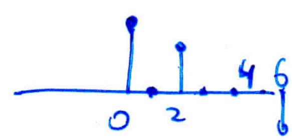


[6 pts] c. Sketch $X_d(e^{j\omega})$:

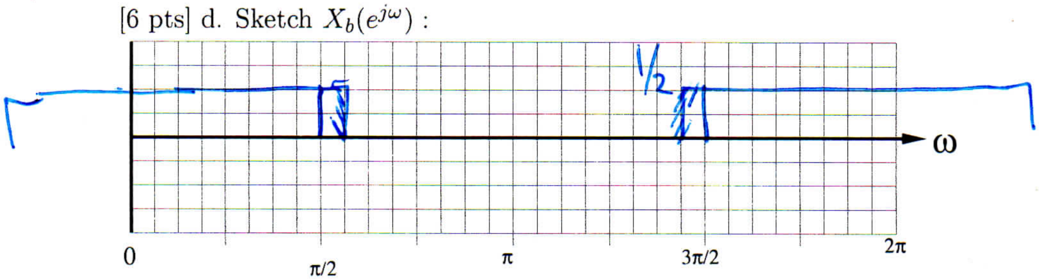
$X_d(e^{j\omega}) = X(e^{j\omega}) \circledast \frac{1}{2\pi} P(e^{j\omega})$



$P(e^{j\omega}) =$ 



[6 pts] d. Sketch $X_b(e^{j\omega})$:



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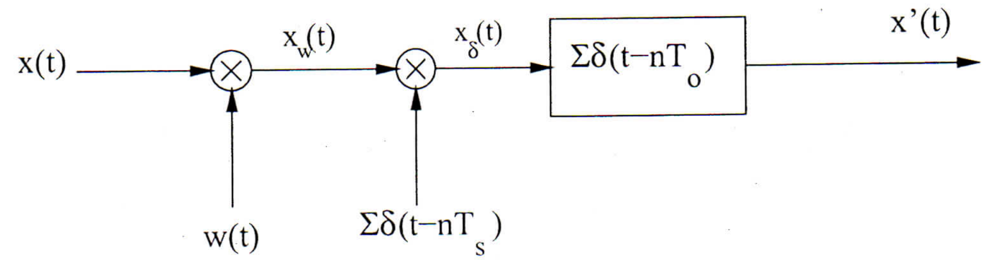
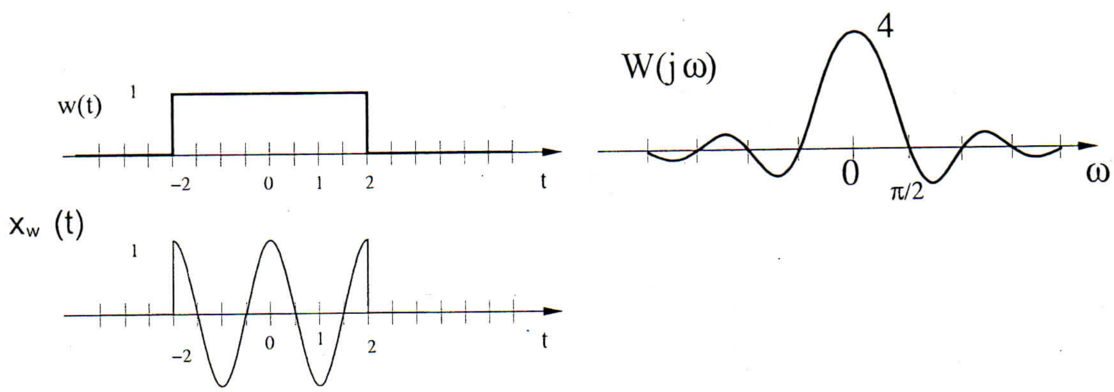
Problem 4. Sampling and Discrete Fourier Transform (30 pts)

Consider the system below, where $x(t) = \cos(\pi t)$. Let $T_s = 0.5$ sec, $T_o = 8$ sec, $w(t) = \Pi(t/4)$. Sketches should label peak magnitudes, and frequency of zero crossing(s) should match given scale.
(All time signals are real and even, hence all spectra are also real and even.)
Note $\Pi(t) = u(t+0.5) - u(t-0.5)$.
Note that the window has spectrum $W(j\omega) = \frac{2\sin 2\omega}{\omega}$.



The window function $w(t)$, windowed cosine $x_w(t)$ and $W(j\omega)$ are shown for convenience here:



$$X_w(j\omega) = W(j\omega) * \frac{1}{2\pi} X(j\omega)$$

$$X_\delta(j\omega) = \frac{1}{2\pi} X_w(j\omega) * \frac{2\pi}{T_s} \sum \delta\left(\omega - \frac{k2\pi}{T_s}\right)$$

$$= X_w(j\omega) * 2 \sum \delta(\omega - k4\pi)$$

$$X'(j\omega) = X_\delta(j\omega) \cdot \underset{\uparrow \atop \pi/4}{\frac{2\pi}{8}} \sum \delta\left(\omega - \frac{k\pi}{4}\right)$$

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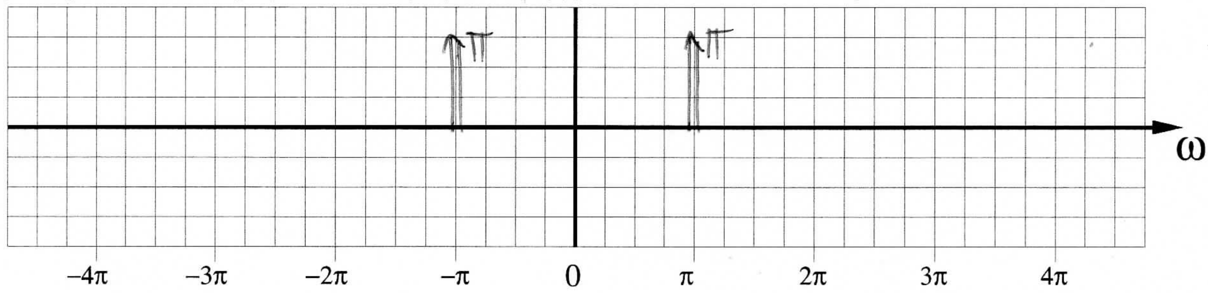
Problem 4. cont.

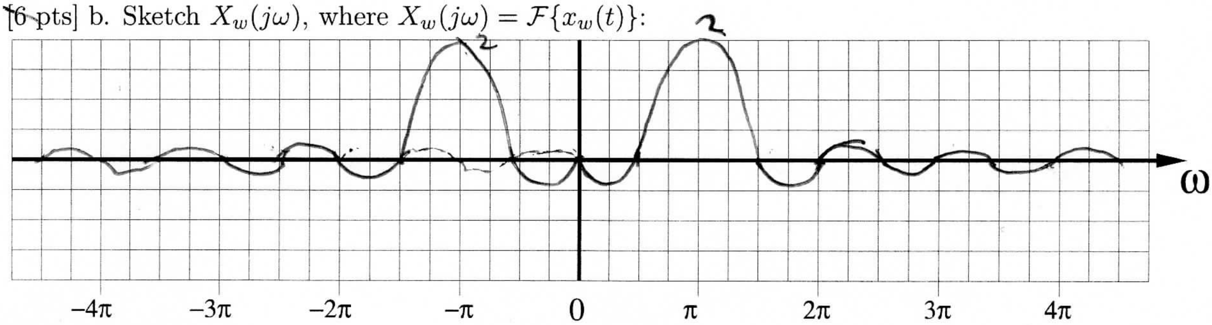
[2 pts] a. Sketch $X(j\omega)$, where $X(j\omega) = \mathcal{F}\{x(t)\}$:



6π
5π

8

[~~6~~ pts] b. Sketch $X_w(j\omega)$, where $X_w(j\omega) = \mathcal{F}\{x_w(t)\}$:



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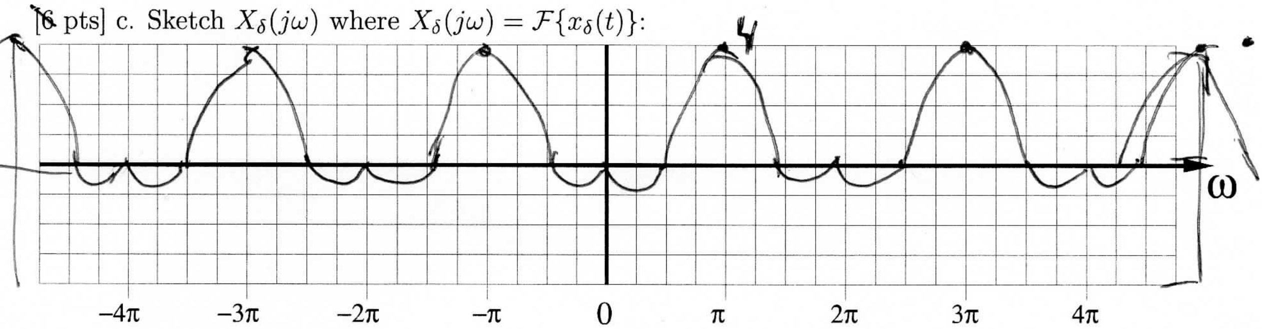
[~~6~~ pts] c. Sketch $X_\delta(j\omega)$ where $X_\delta(j\omega) = \mathcal{F}\{x_\delta(t)\}$:



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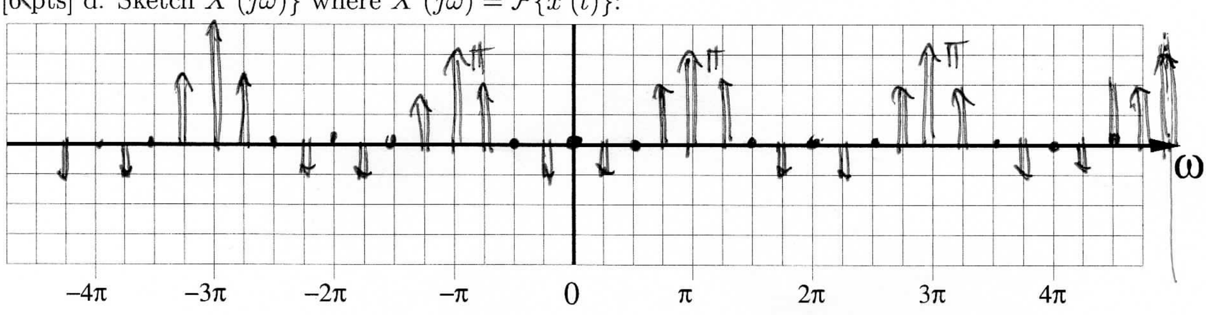
[~~6~~ pts] d. Sketch $X'(j\omega)\}$ where $X'(j\omega) = \mathcal{F}\{x'(t)\}$:



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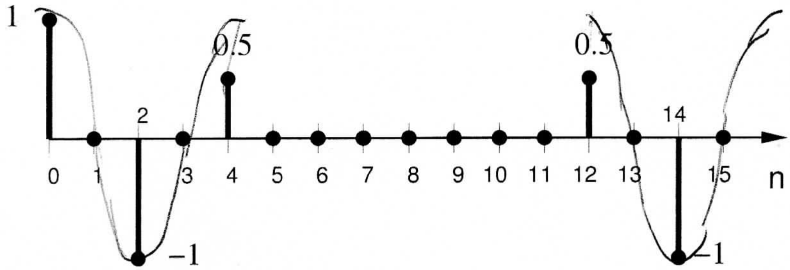
Problem 4. cont.

~~[7 pts]~~ 4 e. Given $x[n] = w[n]\cos(\frac{\pi n}{2})$ as shown below, where $w[n]$ is a window function. $X[k]$ is the 16 point DFT of $x[n]$.

Given $x[n]$:



[2 pts] What is the relation between $x[n]$ and $x'(nT_s)$? $x[n] = \text{area}\{x'(nT_s)\}$

4

[2 pts] What is the relation between $X[4]$ and $X'(j\pi)$? $T_0 = 8$ sec

$$X[k] = \frac{T_0}{2\pi}\,\text{area}\left\{X'\left(jk\frac{2\pi}{T_0}\right)\right\} = \frac{8}{2\pi}\,\text{area}\left\{X'\left(jk\frac{2\pi}{8}\right)\right\}$$

$$X[4] = \frac{4}{\pi}\,\text{area}\{X'(j\pi)\} = \frac{4}{\pi}\cdot\text{area}\{\pi\delta(\omega-\pi)\} = 4.$$

Also

$$X[4] = \sum_{n=0}^{15} x[n]e^{-j8\pi n/16} = \sum_{n=0}^{15} x[n]e^{-j\pi n/2}$$

$$= x[0] - 1(\underbrace{e^{-j2\pi/2} + e^{-j14\pi/2}}_{e^{-j\pi}+e^{-j7\pi} = -2}) + \frac{1}{2}(\underbrace{e^{-j\pi 4/2} + e^{-j\pi 12/2}}_{e^{-j2\pi}+e^{-j6\pi} = 2})$$

$$= 1 + 2 + 1 = 4$$