Phys 7C, Fall 2003, Sect. 1
MT 2 Solutions
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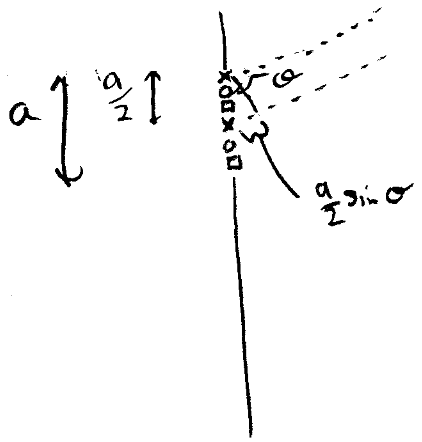
1) a) Using Huygens principle, radiators a <u>half</u> slit width apart will interfere with each other.



X, O, □ are Huygen's wavelets. Note that since I used half the slit width, I can just go down until the upper wavelet is at the middle of the slit, and the lower wavelet is at the bottom of the slit. This justifies using $a/2$ in the derivation.

$$\underbrace{\tfrac{a}{2} \sin \underbrace{\theta_{min,1}}_{\text{angle of 1st minimum}}}_{\text{path length difference}} = \underbrace{\lambda/2}_{\text{half wavelength}}$$

$$a \sin \theta_{min,1} = \lambda$$

$\sin\theta_{min,1} \simeq \theta_{min,1}$ since $y \sim nm$, $L \sim 10\,cm$

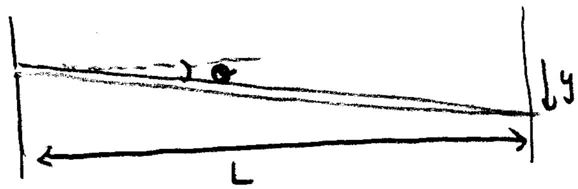
$$a\theta_{min,1} \simeq \lambda$$

$$\boxed{\theta_{min,1} \simeq \lambda/a}$$

b) $\theta_{min,1} \simeq y_{min,1}/L$

$\Rightarrow y_{min,1}/L = \lambda/a$

$y_{min,1} = \lambda L/a$



This is the dist from central max to 1st min on one side.
Dist. between min is twice that.

$$\Delta y_{min,1} = 2 y_{min,1} = 2\lambda L/a = \Delta y_{min,1}$$

$$\Rightarrow a = 2\lambda L/\Delta y_{min,1} = \frac{2(589.3\times10^{-9}\,m)(50\times10^{-2}\,m)}{5.0\times10^{-9}\,m} = 117.86\,m$$

$$\boxed{a = 120\ m}$$

2) a) Lorentz transformation:

$$\Delta t' = \gamma(\Delta t - \tfrac{v}{c^2}\Delta x) = \gamma\left((t_2 - t_1) - \tfrac{v}{c^2}(x_2 - x_1)\right)$$

$$0 = \gamma\left((-\tfrac{1}{2}\text{yr}) - \tfrac{v}{c^2}(1\, c\cdot\text{yr})\right)$$
$$= -\gamma\left(\tfrac{1}{2} + \tfrac{v}{c}\right)\text{yr}$$

$$v/c = -1/2$$

$$\boxed{v = -0.5c} = -1.5\times10^{8}\ \text{m/s}$$

Note: origins coincide @ t=0 thus the frames are moving away from each other. The origin of the S' frame is moving in the negative x-dir, but it is still moving away from the S origin

b) $$\Delta t' = \gamma(\Delta t - \tfrac{v}{c^2}\Delta x)$$
$$(t_1' - 0) = \gamma\left((t_1 - 0) - \tfrac{v}{c^2}(x_1 - 0)\right)$$

clocks synchronized @ t=0

$$t_1' = \gamma(t_1 - \tfrac{v}{c^2}x_1)$$
$$= \frac{1}{\sqrt{1-(v/c)^2}}\left(1\,\text{yr} - \tfrac{v}{c^2}(1\,c\cdot\text{yr})\right)$$
$$= \frac{(1 - v/c)\,\text{yr}}{\sqrt{1-(v/c)^2}} = \frac{1+\frac{1}{2}}{\sqrt{1-(1/2)^2}} = \frac{3/2}{\sqrt{3/4}} = \frac{2\cdot\frac{3}{2}}{\sqrt{3}}$$
$$= 3/\sqrt{3}$$

$$= \boxed{\sqrt{3}\,\text{yr} = t_1' = 1.73\,\text{yr}} = 5.5\times10^{7}\ \text{sec}$$

The events are simultaneous in S'

$$\Rightarrow \boxed{t_2' = \sqrt{3}\,\text{yr} = 1.73\,\text{yr}}$$

3) a) Work function $W = 1.85 eV$ $(2.96 \times 10^{-19} J)$

Max kinetic energy of ejected electron:

$$E_{k,max} = h\nu - W$$

Where $h\nu$ is the energy of the incident photon.

$$E_{k,max} = \frac{hc}{\lambda} - W = \frac{1240 \text{ eV}\cdot\text{nm}}{\lambda} - 1.85 eV$$

$$= \frac{1240 \text{ eV}\cdot\text{nm}}{410 \text{ nm}} - 1.85 eV = 3.02 eV - 1.85 eV$$

$$E_{k,max} = 1.17 eV \quad (= 1.88 \times 10^{-19} J)$$

An electron th' crosses potential ϕ has potential energy $e\phi$ When this potential energy is equal to its initial kinetic energy, the electron will stop. This is the stopping potential.

$$e\phi_{stop} = E_{k,max} = h\nu - W = 1.17 eV$$

$$\Rightarrow \phi_{stop} = \frac{E_{k,max}}{e} = \frac{h\nu - W}{e} = \frac{1.17 eV}{e}$$

$$\boxed{\phi_{stop} = 1.17 V}$$

b) $E_{k,max} = 1.17 eV \ll m_e c^2 = 0.511 MeV$

Thus the electron is nonrelativistic. ← You must check this to state this

$\underline{E_k = \frac{1}{2}mv^2} = 1.17 eV$ (-5 for not checking)

$$v = \sqrt{\frac{2E_k}{m}} = c\sqrt{\frac{2E_k}{mc^2}} = c\sqrt{\frac{2 \cdot 1.17\,eV}{0.511 \times 10^6\,eV}}$$

$$\boxed{v = 2.14 \times 10^{-3}\,c = 6.42 \times 10^5\ m/s}$$

note - an electron can go 642 km in one second
~ Berkeley to Long Beach in one second
without being relativistic!

4) a) $L = n\hbar = m v_n r_n$ quantization of angular momentum (given)

Assume planetary system where the electron is orbiting the proton. This time, however, the potential between the electron and the proton is:

$$V(r) = Cr^4$$

As opposed to $V(r) = e/4\pi\varepsilon_0 r$, as is in the Coulomb case.

The Force between the e^- & the p^+ is then

$$F = \text{"}-e\frac{\partial V}{\partial r}\text{"} = -4Cer^3$$ note negative. Inward, $(-\hat{r})$.

↑ stated by Ilya during the exam.

This must be equal to the centripetal acceleration of the electron. Acceleration, too, is inward. $(-\hat{r})$

$$-4Cer_n^3 = -\frac{mV_n^2}{r_n}$$ (-3 for sign error)

Solving for V_n

$$V_n = \sqrt{\frac{4cer_n^4}{m}} = 2r_n^2\sqrt{\frac{Ce}{m}}$$

The potential energy (not potential) of an electron is just $E_{pot} = -eV$

Thus the total energy of the electron in orbit is:

$$E_n = E_{kin} + E_{pot} = \frac{1}{2}mV_n^2 - eCr_n^4$$

plugging in for V_n

$$E_n = \frac{1}{2} m \left(2 r_n^2 \sqrt{\frac{Ce}{m}} \right)^2 - eCr_n^4$$

$$= 2eCr_n^4 - eCr_n^4 = eCr_n^4 = E_n \quad *$$

From the quantization of angular momentum:

$$L = n\hbar = m V_n r_n$$

$$n\hbar = m r_n \left(2 r_n^2 \sqrt{\frac{Ce}{m}} \right) = 2 r_n^3 \sqrt{Cme}$$

$$\Rightarrow \quad r_n = \left[\frac{n\hbar}{2\sqrt{Cme}} \right]^{1/3}$$

plugging this into (*)

$$E_n = eC \left[\frac{n\hbar}{2\sqrt{Cme}} \right]^{4/3} = R\, n^{4/3}$$

where $\boxed{R \equiv \frac{C^{1/3} e^{1/3} \hbar^{4/3}}{2^{4/3} m^{2/3}}}$

b) note, $r_n \propto n^{1/3}$

$$\Rightarrow \quad \frac{r_{n'}}{r_n} = \left(\frac{n'}{n}\right)^{1/3}$$

In this case $r_{n'} = 3a$, $r_n = r_1 = a$

$$\frac{3a}{a} = \left(\frac{n'}{1}\right)^{1/3}$$

$$3 = (n')^{1/3}$$

$$\Rightarrow \quad n' = 3^3 = 27$$

$$\boxed{n' = 27}$$