Physics 7B

Midterm 1: Monday September 26th, 2016

Instructors: Prof. R.J. Birgeneau/Dr. A. Frano

Total points: 100 (5 problems)

Show all your work and take particular care to explain what you are doing. Partial credit can be given. Please use the symbols described in the problems or define any new symbol that you introduce. Label any drawings that you make. ***Good luck!***

Problem 1 (20 pts)

Consider an ideal gas of N diatomic molecules, which have 5 degrees of freedom.

(a) By the equipartition theorem, what is the internal energy of the gas E_{int}, and what is its molar specific heat at constant volume, C_V?

(b) Recalling the first law of thermodynamics ($dE_{int} = TdS - PdV$), find the change in entropy, ΔS, which results from isovolumetrically increasing the temperature from T_i to T_f.

(c) Consider an adiabatic expansion from volume V_i to V_f, over which the temperature changes from T_i to T_f and the pressure changes from P_i to P_f. Use the first law and the ideal gas equation of state to show that $P_i V_i^\gamma = P_f V_f^\gamma$ and give an expression for γ.

Problem 2 (20 pts)

An ideal gas starts out in a state A, with pressure P_A, volume V_A, and temperature T_A. First, the gas expands isothermally until its volume has tripled and it reaches point B. Then, the gas cools isovolumetrically to reach point C, at which point it is adiabatically compressed to return to point A. This gas has $\gamma = \frac{C_P}{C_V} = 5/3$. Make sure to express your answers in terms of only the variables stated in this problem.

(a) Draw a PV diagram for this cycle.

(b) What is the work done by the gas during each step of this cycle?

(c) What is the heat flowing into the gas during each step of this cycle?

(d) What is the change in entropy in the gas during each step of this cycle?

Problem 3 (20 pts)

The Carnot cycle comprises four segments: (1) isothermal expansion from $A \to B$ with heat transfer Q_H at T_H, (2) adiabatic expansion from $B \to C$, (3) isothermal compression from $C \to D$ at T_L with heat transfer $-Q_L$, and (4) adiabatic compression from $D \to A$.

(a) Sketch the cycle in a PV diagram.

(b) Derive the efficiency of this cycle, defined as $e = \frac{W}{Q_H}$ in terms of the two temperatures T_H and T_L? Show all your work throughout the derivation.

Problem 4 (20 pts)

Suppose N gas molecules each with mass m live in a very narrow cylinder, meaning they can only move in 1 dimension. The length of the cylinder is "L". The gas particles do not interact with each other, but when they reach the end walls (the walls separated by "L") they elastically collide and change direction.

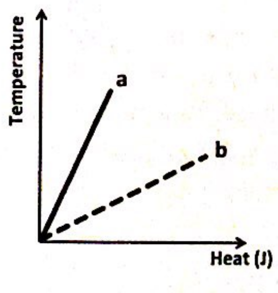
(a) If a molecule can have velocity $\vec{v} = \hat{x}v_x$ or $\vec{v} = -\hat{x}v_x$, how much time would go by between collisions on a given wall?

(b) How much force F needs to be applied at each wall to balance the force exerted by the gas molecules? (Assume all molecules behave like in (a).)

(c) Using the temperature definition $T = mv^2/k_B$, derive the equation of state of the gas.

Problem 5 (20 pts)

(a) The figure below shows a plot of the temperature of two materials **a** (solid) and **b** (dotted) as function of added heat. Which material has higher specific heat?



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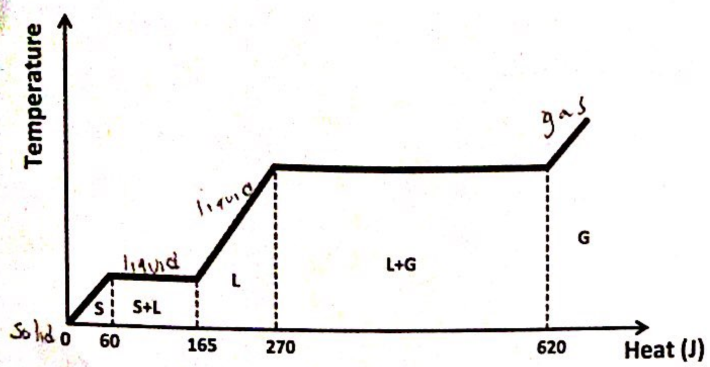
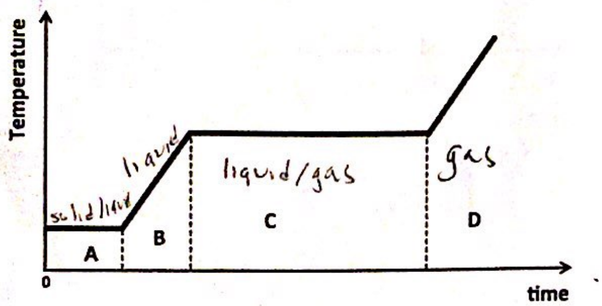
(b) The figure below shows a plot of the temperature of 1 kg of a material as function of added heat (J). What are the heat of fusion L_f and heat of vaporization L_v of the material?



(c) The figure below shows a plot of the temperature of a material from some initial state, as function of time with heat added at a constant rate. The different regions are labeled A, B, C, D. Which of these regions correspond to mixed phases? Which region corresponds to a solid-liquid transition? Which region corresponds to a liquid-gas transition? Can region A span a longer time than region C? Explain.



(d) 1 kg of solid mercury is at a temperature $-40°C$ (which we take as its melting point) and has specific heat $c_{Hg} = 1400$ J/(kg K). Then, it is put in an aluminum container weighing $m_{Al} = 2$ kg (Al has a specific heat of $c_{Al} = 900$ J/(kg K)) and filled with $m_w = 1$ kg of water at a temperature of $T_w = 50°C$ (take the specific heat of water as $c_w = 4200$ J/(kg K)). The container is isolated from its environment, and at equilibrium the temperature of the system is $T_f = 30°C$. Calculate the latent heat of fusion of mercury.

Formula Sheet: Physics 7B, Midterm 1 (Fall 2016)

$\Delta E_{int} = Q - W$

$e_{ideal} = 1 - \frac{T_C}{T_H}$

$COP_r = \frac{Q_C}{W_{net}}$

Thermodynamics

$$\Delta l = \alpha l_0 \Delta T$$

$$\Delta V = \beta V_0 \Delta T$$

$$Q = mc\Delta T = nC\Delta T$$

$$Q = mL \text{ (For a phase transition)}$$

$$C_P - C_V = R = N_A k_B$$

$$\frac{dQ}{dt} = -kA\frac{dT}{dx}$$

$$e = \frac{W_{net}}{Q_{in}}$$

$$v_{rms} = \sqrt{\frac{3k_BT}{m}} \text{ (for a monatomic gas)}$$

$$e = \frac{W_{net}}{Q_{in}}$$

$$\Delta S = \int \frac{dQ}{T} \text{ (For reversible processes)}$$

$$dQ = TdS$$

$$\Delta S_{syst} + \Delta S_{env} > 0$$

$$\oint dS = 0$$

Calculus

$$\overline{g(v)} = \int_0^\infty \frac{g(v)f(v)}{N} dv$$

($f(v)$ a speed distribution)

$$\int_0^\infty x^n e^{-ax} dx = \frac{n!}{a^{n+1}}$$

$$\int_0^\infty x^{2n} e^{-ax^2} dx = \frac{(2n)!}{n!\,2^{2n+1}}\sqrt{\frac{\pi}{a^{2n+1}}}$$

$$\int_0^\infty x^{2n+1} e^{-ax^2} dx = \frac{n!}{2a^{n+1}}$$

$$\int (1+x^2)^{-\frac{1}{2}} dx = \ln(x + \sqrt{1+x^2})$$

$$\int (1+x^2)^{-1} dx = \arctan(x)$$

$$\int (1+x^2)^{-\frac{3}{2}} dx = \frac{x}{\sqrt{1+x^2}}$$

$$\int \frac{x}{1+x^2} dx = \frac{1}{2}\ln(1+x^2)$$

$$\int \frac{1}{\cos(x)} dx = \ln\left(\left|\tan\left(\frac{x}{2}+\frac{\pi}{4}\right)\right|\right)$$

$$\int \frac{1}{\sin(x)} dx = \ln\left(\left|\tan\left(\frac{x}{2}\right)\right|\right)$$

$$\sin(x) \approx x$$

$$\cos(x) \approx 1 - \frac{x^2}{2}$$

$$e^x \approx 1 + x + \frac{x^2}{2}$$

$$(1+x)^\alpha \approx 1 + \alpha x + \frac{\alpha(\alpha-1)}{2}x^2$$

$$\ln(1+x) \approx x - \frac{x^2}{2}$$

$$\sin(2x) = 2\sin(x)\cos(x)$$

$$\cos(2x) = 2\cos^2(x) - 1$$

$$\sin(a+b) = \sin(a)\cos(b) + \cos(a)\sin(b)$$

$$\cos(a+b) = \cos(a)\cos(b) - \sin(a)\sin(b)$$

$$1 + \cot^2(x) = \csc^2(x)$$

$$1 + \tan^2(x) = \sec^2(x)$$