MIDTERM 1 Fall-2016

Instructor: Prof. A. LANZARA

TOTAL POINTS: 100

Show all work, and take particular care to explain what you are doing. Partial credit is given. Please use the symbols described in the problems, define any new symbol that you introduce and label any drawings that you make. All answers should be in terms of variables.
If you get stuck, skip to the next problem and return to the difficult section later in the exam period.

PROBLEM 1 (Tot 20pts)

An adiabatic cylinder of volume V_1 contains a mixture of two chemically inert gases. The mixture is formed by n_1 moles of an ideal monoatomic gas of specific heat C_{V1} and n_2 moles of an ideal diatomic gas of specific heat C_{V2}. The mixture is in equilibrium at temperature T_1. The gas mixture then expands to twice its initial volume in an adiabatic and reversible way.

a) (10pts) Find the molar specific heat C_V of the mixture.
b) (10pts) Find the final equilibrium temperature of the system.

PROBLEM 2 (Tot 20pts)

One mole of a ideal monoatomic gas undergoes the following cycle:
AB – reversible isotherm at temperature T_A
BC – reversible isobaric at pressure P_B
CA – irreversible isovolumetric that bring the system back to temperature T_A by exchanging the right amount of heat.
If $V_B=2V_C$ with $V_C=1m^3$ and $P_A=2P_B$:

a) (5pts) Draw the cycle in the PV diagram.
b) (8pts) Find the work done by the gas during the cycle. Is this work positive or negative? Explain your answer.
c) (7pts) Find the heat exchanged with the environment in each of the three processes and express it in terms of the known variables (temperature and pressure).

PROBLEM 3 (Tot 15pts)

A container, thermodynamically isolated, contains a mass M of ice at temperature T=0°C. An equivalent mass M of water is now added to the container. The temperature of the water is T_W. The pressure of the water + ice system is P, the specific heat of water is C_W, and the latent heat of fusion of the ice is L. The total volume of the water + ice system does not change when the ice melt into water. Neglect the specific heat of the container.
Find:

1) (5pts) The variation of internal energy of the system water + ice, between the initial and final state.

2) (10pts) If all the ice melts, find the final equilibrium temperature of the water + ice system.

PROBLEM 4 (15pts).

Two identical containers contain two different diatomic gasses. While the total mass of the gas in each container is the same, the total number of molecules in A is N_A at pressure P_A and the total number of molecules in B is N_B at pressure P_B. The two gases are at the same temperature, T.

a) (5pts) What is the ratio of RMS velocities, of the gas in A and B?

b) (10pts) If we want $v^A_{RMS} = v^B_{RMS}$, by what fraction $(T_A - T) / T$ should the new temperature T_A, in container A, be changed from T?

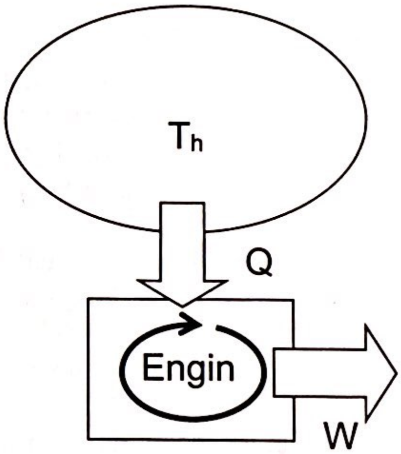
PROBLEM 5 (Tot 30pts)

PART I) (10 pts)

A thermodynamically isolated cylinder of volume V contains an ideal gas. The gas undergoes a free expansion. The final volume is twice the initial volume. Is the change in entropy positive, negative or zero? Show your answer step by step.

PART II) (10 pts)

In one complete cycle, a heat engine extracts heat Q from a thermal reservoir, does work W and does **not** eject any heat into the environment. Is this a possible engine? Explain your answer and show each step of your reasoning.



PART III) (10pts)

n moles of a monatomic ideal gas undergo the following cycle: starting from P_0 at temperature T_0, it contracts adiabatically to $P = 2P_0$, then it expands isothermally, then it contracts isobarically.

Draw this cycle in the PV diagram (3pts) and determine its efficiency (7pts)?

Formula Sheet: Physics 7B, Midterm 1 (Fall 2016)

Thermodynamics

$$\Delta l = \alpha l_0 \Delta T$$

$$\Delta V = \beta V_0 \Delta T$$

$$Q = mc\Delta T = nC\Delta T$$

$Q = mL$ (For a phase transition)

$$C_P - C_V = R = N_A k_B$$

$$\frac{dQ}{dt} = -kA\frac{dT}{dx}$$

$$e = \frac{W_{net}}{Q_{in}}$$

$v_{rms} = \sqrt{\frac{3k_BT}{m}}$ (for a monatomic gas)

$$e = \frac{W_{net}}{Q_{in}}$$

$\Delta S = \int \frac{dQ}{T}$ (For reversible processes)

$$dQ = TdS$$

$$\Delta S_{syst} + \Delta S_{env} > 0$$

$$\oint dS = 0$$

Calculus

$$\overline{g(v)} = \int_0^\infty \frac{g(v)f(v)}{N}dv$$

($f(v)$ a speed distribution)

$$\int_0^\infty x^n e^{-ax} dx = \frac{n!}{a^{n+1}}$$

$$\int_0^\infty x^{2n} e^{-ax^2} dx = \frac{(2n)!}{n!\,2^{2n+1}}\sqrt{\frac{\pi}{a^{2n+1}}}$$

$$\int_0^\infty x^{2n+1} e^{-ax^2} dx = \frac{n!}{2a^{n+1}}$$

$$\int (1+x^2)^{-\frac{1}{2}} dx = \ln(x + \sqrt{1+x^2})$$

$$\int (1+x^2)^{-1} dx = \arctan(x)$$

$$\int (1+x^2)^{-\frac{3}{2}} dx = \frac{x}{\sqrt{1+x^2}}$$

$$\int \frac{x}{1+x^2} dx = \frac{1}{2}\ln(1+x^2)$$

$$\int \frac{1}{\cos(x)} dx = \ln\left(\left|\tan\left(\frac{x}{2}+\frac{\pi}{4}\right)\right|\right)$$

$$\int \frac{1}{\sin(x)} dx = \ln\left(\left|\tan\left(\frac{x}{2}\right)\right|\right)$$

$$\sin(x) \approx x$$

$$\cos(x) \approx 1 - \frac{x^2}{2}$$

$$e^x \approx 1 + x + \frac{x^2}{2}$$

$$(1+x)^\alpha \approx 1 + \alpha x + \frac{\alpha(\alpha-1)}{2}x^2$$

$$\ln(1+x) \approx x - \frac{x^2}{2}$$

$$\sin(2x) = 2\sin(x)\cos(x)$$

$$\cos(2x) = 2\cos^2(x) - 1$$

$$\sin(a+b) = \sin(a)\cos(b) + \cos(a)\sin(b)$$

$$\cos(a+b) = \cos(a)\cos(b) - \sin(a)\sin(b)$$

$$1 + \cot^2(x) = \csc^2(x)$$

$$1 + \tan^2(x) = \sec^2(x)$$