Math 16A Second Midterm 6 Nov 2008

NAME (1 pt): ______________________

TA (1 pt): ______________________

Name of Neighbor to your left (1 pt): ______________________

Name of Neighbor to your right (1 pt): ______________________

Instructions: This is a closed book, closed notes, closed calculator, closed phone, closed computer, closed network, open brain exam.

You get one point each for filling in the 4 lines at the top of this page. If you are sitting next to the steps or the wall, "steps" or "wall" are acceptable answers for the last 2 lines. Fill in these lines, and then wait for us to tell you to turn the page and start.

All other questions are worth the number of points shown. You may only use techniques in the book through the end of Section 4.3 in order to solve the questions. For full (or partial) credit you need to show your work. You may of course check your answers using any other facts you know.

Write all your answers on this exam. If you need scratch paper, ask for it, write your name on each sheet, and attach it when you turn it in (we have a stapler).

At the end of the exam, you must line up and turn in your exam personally to your Teaching Assistant. Have your photo ID out so that the Teaching Assistant can look at it. You must have your photo ID in order to turn in your exam.

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Total	

ANSWERS

Question 1. (15 points) Mark the following statements "True" or "False".
(DO NOT GUESS: -2 points for each wrong answer!)

(a) $\frac{d}{dx}\frac{e^{2x}+2e^x+1}{e^x+1} = e^x$ T F
Answer: True (the fraction simplifies to $e^x + 1$).

(b) $\lim_{x\to 0}\frac{\sqrt{e^{9x}}-1}{x} = 3$ T F
Answer: False (the limit equals $\frac{9}{2}$).

(c) $\frac{d}{dy}h(g(f(y))) = h'(g(f(y)))g'(f(y))f'(y)$ T F
Answer: True (use the Chain Rule twice).

(d) If $f(x)$ is differentiable for $0 \le x \le 1$, and $f(0) = f(1) = 2$,
then there is some $0 \le a \le 1$ such that $f'(a) = 0$. T F
Answer: True (draw a picture; if the function goes up and then back down, or down and then up, it will have an extreme point at which $f'(a) = 0$; or it could be constant $f(x) = 2$ for all x, so $f'(x) = 0$ for all x).

(e) The function $f(x) = x^3 + 4x^2 + 10x - 828$ has 2 local extrema. T F
Answer: False (if you try to find the roots of $f'(x) = 0$ using the quadratic formula, you get the square root of a negative number).

Question 1. (15 points) Mark the following statements "True" or "False".
(<u>DO NOT GUESS</u>: -2 points for each wrong answer!)

(a) $\lim_{y\to 0} \frac{\sqrt{e^{16y}}-1}{y} = 4$ T F
Answer: False (the limit equals $\frac{16}{2}$).

(b) $\frac{d}{dy}\frac{e^{2y}-2e^y+1}{e^y-1} = e^y$ T F
Answer: True (the fraction simplifies to $e^y - 1$).

(c) $\frac{d}{dy} g(f(h(y))) = g'(f(h(y)))f'(h(y))h'(y)$ T F
Answer: True (use the Chain Rule twice).

(d) The function $g(y) = y^3 + 3y^2 + 8y + 962$ has 2 local extrema. T F
Answer: False (if you try to find the roots of $g'(y) = 0$ using the quadratic formula, you get the square root of a negative number).

(e) If $g(y)$ is differentiable for $2 \le y \le 3$, and $g(2) = g(3) = -1$,
then there is some $2 \le a \le 3$ such that $g'(a) = 0$. T F
Answer: True (draw a picture; if the function goes up and then back down, or down and then up, it will have an extreme point at which $g'(a) = 0$; or it could be constant $g(y) = -1$ for all y, so $g'(y) = 0$ for all y).

Question 1. (15 points) Mark the following statements "True" or "False".
(<u>DO NOT GUESS</u>: -2 points for each wrong answer!)

(a) $\frac{d}{dz}h(f(g(z))) = h'(f(g(z)))f'(g(z))g'(z)$ T F
Answer: True (use the Chain Rule twice).

(b) $\lim_{z\to 0} \frac{\sqrt{e^{25z}}-1}{z} = 5$ T F
Answer: False (the limit equals $\frac{25}{2}$).

(c) $\frac{d}{dz}\frac{e^{2z}-1}{e^z+1} = e^z$ T F
Answer: True (the fraction simplifies to $e^z - 1$).

(d) The function $h(z) = z^3 + 5z^2 + 11z - 187$ has 2 local extrema. T F
Answer: False (if you try to find the roots of $h'(z) = 0$ using the quadratic formula, you get the square root of a negative number).

(e) If $h(z)$ is differentiable for $-1 \leq z \leq 1$, and $h(-1) = h(1) = -2$,
then there is some $-1 \leq a \leq 1$ such that $h'(a) = 0$. T F
Answer: True (draw a picture; if the function goes up and then back down, or down and then up, it will have an extreme point at which $h'(a) = 0$; or it could be constant $h(z) = -2$ for all z, so $h'(z) = 0$ for all z).

Question 2. (16 points)

- (a) For $f(x) = -\frac{x}{(2x+3)^2}$, find $f'(x)$ (**in simplified form**), and determine if $f(x)$ is increasing or decreasing at $x = -2$.
- (b) Determine all critical points of $y = x^6 e^{-3x^2}$. (You must again compute $\frac{dy}{dx}$ correctly in a form that makes this determination possible)

Answer:

- *(a) For $f(x) = -\frac{x}{(2x+3)^2}$, find $f'(x)$ (**in simplified form**), and determine if $f(x)$ is increasing or decreasing at $x = -2$.*

  $f'(x) = -\frac{1\cdot(2x+3)^2 - x\cdot 2(2x+3)\cdot 2}{(2x+3)^4} = -\frac{(2x+3)[2x+3-4x]}{(2x+3)^4} = -\frac{-2x+3}{(2x+3)^3}$
  Since $f'(-2) = -\frac{7}{(-1)^3} = 7 > 0$, f is increasing at $x = -2$

- *(b) Determine all critical points of $y = x^6 e^{-3x^2}$. (You must again compute $\frac{dy}{dx}$ correctly in a form that makes this determination possible)*

  $$\frac{dy}{dx} = (6x^5)e^{-3x^2} + x^6(e^{-3x^2}(-6x)) = -6x^5 e^{-3x^2}(x^2 - 1)$$

  Setting this equal to 0, and noting that $e^{-3x^2} > 0$, we find 3 critical points; $x = 0$, and $x = \pm 1$

Question 2. (16 points)

- (a) For $g(x) = -\frac{x}{(3x+5)^2}$, find $g'(x)$ (**in simplified form**), and determine if $g(x)$ is increasing or decreasing at $x = -2$.
- (b) Determine all critical points of $y = x^8 e^{-2x^2}$. (You must again compute $\frac{dy}{dx}$ correctly in a form that makes this determination possible)

Answer:

- *(a) For $g(x) = -\frac{x}{(3x+5)^2}$, find $g'(x)$ (**in simplified form**), and determine if $g(x)$ is increasing or decreasing at $x = -2$.*
  $g'(x) = -\frac{1\cdot(3x+5)^2 - x\cdot 2(3x+5)\cdot 3}{(3x+5)^4} = -\frac{(3x+5)[3x+5-6x]}{(3x+5)^4} = -\frac{-3x+5}{(3x+5)^3}$
  Since $g'(-2) = -\frac{11}{(-1)^3} = 11 > 0$, g is increasing at $x = -2$
- *(b) Determine all critical points of $y = x^8 e^{-2x^2}$. (You must again compute $\frac{dy}{dx}$ correctly in a form that makes this determination possible)*

$$\frac{dy}{dx} = (8x^7)e^{-2x^2} + x^8(e^{-2x^2}(-4x)) = -4x^7 e^{-2x^2}(x^2 - 2)$$

Setting this equal to 0, and noting that $e^{-2x^2} > 0$, we find 3 critical points; $x = 0$, and $x = \pm\sqrt{2}$

Question 2. (16 points)

- (a) For $h(x) = \frac{x}{(4x+7)^2}$, find $h'(x)$ (**in simplified form**), and determine if $h(x)$ is increasing or decreasing at $x = -2$.
- (b) Determine all critical points of $y = x^{24}e^{-4x^2}$. (You must again compute $\frac{dy}{dx}$ correctly in a form that makes this determination possible)

Answer:

- *(a) For $h(x) = \frac{x}{(4x+7)^2}$, find $h'(x)$ (**in simplified form**), and determine if $h(x)$ is increasing or decreasing at $x = -2$.*
  $h'(x) = \frac{1\cdot(4x+7)^2 - x\cdot 2(4x+7)\cdot 4}{(4x+7)^4} = \frac{(4x+7)[4x+7-8x]}{(4x+7)^4} = \frac{-4x+7}{(4x+7)^3}$
  Since $h'(-2) = \frac{15}{(-1)^3} = -15 < 0$, h is decreasing at $x = -2$
- *(b) Determine all critical points of $y = x^{24}e^{-4x^2}$. (You must again compute $\frac{dy}{dx}$ correctly in a form that makes this determination possible)*

$$\frac{dy}{dx} = (24x^{23})e^{-4x^2} + x^{24}(e^{-4x^2}(-8x)) = -8x^{23}e^{-4x^2}(x^2 - 3)$$

Setting this equal to 0, and noting that $e^{-2x^2} > 0$, we find 3 critical points; $x = 0$, and $x = \pm\sqrt{3}$

Question 3. (16 points)

(a) If $f(x) = x^2 - 2$ and $g(x) = x^{3/2}$, compute $\frac{d}{dx}(f(g(x)))$ by the CHAIN RULE.

Answer: $\frac{d}{dx}(f(g(x))) = f'(g(x)) \cdot g'(x)$.
We know $f'(x) = 2x$ *and* $g'(x) = \frac{3}{2}\sqrt{x}$
so our desired answer is $3x^2$

(b) Now, instead consider $k(x) = h(x^2 - 2) \cdot x^{3/2}$, where $h(\ )$ is a function. Find an expression for $k'(x)$.

Answer: The desired derivative is $(3/2)h(x^2 - 2)x^{1/2} + (2x^{5/2})h'(x^2 - 2)$

(c) Let's consider a rectangular box whose edges have lengths x, $2x$, and y, so that its surface area is $6xy + 4x^2$. We constrain the surface area to be 28 square units. The lengths of the sides (in terms of x and y) are changing with time (t in seconds). At what rate is y changing when $y = 1, x = 2$, and x is increasing 6 *units/second* [i.e. $\frac{dx}{dt} = 6$]?

Answer: $\frac{d}{dt}\left(28 = 6xy + 4x^2\right)$ *gives us*
$0 = 6x\frac{dy}{dt} + 6y\frac{dx}{dt} + 8x\frac{dx}{dt} = \frac{dx}{dt}(6y + 8x) + \frac{dy}{dt}(6x)$.
We want the change in y *over time, so* $\frac{dy}{dt} = -\frac{dx}{dt}\frac{6y+8x}{6x}$.
Substituting the given values, $\frac{dy}{dt} = -(6)\frac{6(1)+8(2)}{6(2)} = (-6)\frac{22}{12} = -11$ *units/second.*

Question 3. (16 points)

(a) If $f(x) = 3x + 1$ and $g(x) = x^{3/2}$, compute $\frac{d}{dx}(f(g(x)))$ by the CHAIN RULE.

Answer: $\frac{d}{dx}(f(g(x))) = f'(g(x)) \cdot g'(x)$.
We know $f'(x) = 3$ *and* $g'(x) = \frac{3x^{1/2}}{2}$
so our desired answer is $\frac{9x^{1/2}}{2}$

(b) Now, instead consider $k(x) = \frac{h(3x+1)}{x^{3/2}}$, where $h(\ )$ is a function. Find an expression for $k'(x)$.

Answer: *The desired derivative is* $\frac{3x^{3/2}h'(3x+1)-(3/2)x^{1/2}h(3x+1)}{x^3}$

(c) Let's consider a rectangular box whose edges have lengths $2x$, $3x$, and y, so that its surface area is $10xy + 12x^2$. We constrain the surface area to be 22 square units. The lengths of sides (in terms of x and y) are changing with time (t in seconds). At what rate is y changing when $x = 1, y = 1$, and x is increasing 5 *units/second* [i.e. $\frac{dx}{dt} = 5$].

Answer: $\frac{d}{dt}\left(22 = 10xy + 12x^2\right)$ *gives us*
$0 = 10x\frac{dy}{dt} + 10y\frac{dx}{dt} + 24x\frac{dx}{dt} = \frac{dx}{dt}(10y + 24x) + \frac{dy}{dt}(10x)$.
We want the change in y *over time, so* $\frac{dy}{dt} = -\frac{dx}{dt}\frac{10y+24x}{10x}$.
Substituting the given values, $\frac{dy}{dt} = -(5)\frac{10(1)+24(1)}{10(1)} = (-5)\frac{34}{10} = -17$ *units/second*

Question 3. (16 points)

(a) If $f(x) = x^2 + 5$ and $g(x) = x^{4/3}$, compute $\frac{d}{dx}(f(g(x)))$ by the CHAIN RULE.

Answer: $\frac{d}{dx}(f(g(x))) = f'(g(x)) \cdot g'(x)$. *We know* $f'(x) = 2x$ *and* $g'(x) = \frac{4}{3} \cdot x^{1/3}$ *so our desired answer is* $\frac{8x^{5/3}}{3}$

(b) Now, instead consider $k(x) = \frac{h(x^{4/3})}{x^2+5}$, where $h(\ )$ is a function. Find an expression for $k'(x)$.

Answer: *The desired derivative is* $\frac{(4/3)(x^2+5)(x^{1/3})h'(x^{4/3})-2xh(x^{4/3})}{(x^2+5)^2}$

(c) Let's consider a rectangular box whose edges have lengths $2x$, $6x$ and y, so that its surface area is $16xy + 24x^2$. We constrain the surface area to be 56 square units. The lengths of sides (in terms of x and y) are changing with time (t seconds). At what rate is y changing when $y = 2, x = 1$, and x is increasing 2 *units/second* [i.e. $\frac{dx}{dt} = 2$]?

Answer: $\frac{d}{dt}\left(56 = 16xy + 24x^2\right)$ *gives us*
$0 = 16x\frac{dy}{dt} + 16y\frac{dx}{dt} + 48x\frac{dx}{dt} = \frac{dx}{dt}(16y + 48x) + \frac{dy}{dt}(16x)$.
We want the change in y *over time, so* $\frac{dy}{dt} = -\frac{dx}{dt}\frac{16y+48x}{16x}$.
Substituting the given values, $\frac{dy}{dt} = -(2)\frac{16(2)+48(1)}{16(1)} = (-2)\frac{80}{16} = -10$ *units/second*

Question 4. (18 points)
Part a.) Sketch the graph of $y = f(x) = -x^3 + 3x^2 + 9x - 5$. You do not need to find the x-intercepts, but all other information about the graph should be filled in the following table:

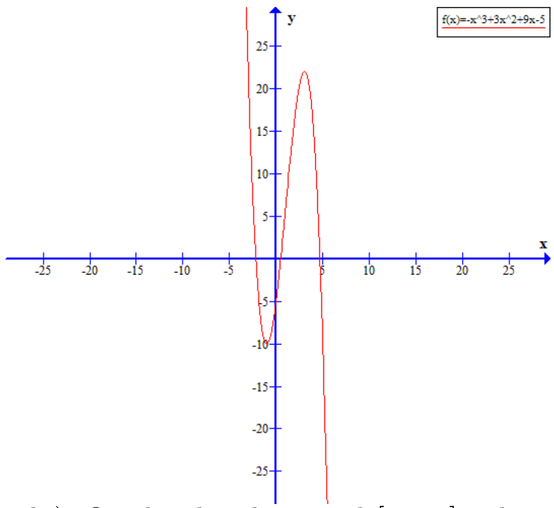
Answer:

critical points (x, y)	$(-1, -10)$, $(3, 22)$
relative maxima (x, y)	$(3, 22)$
relative minima (x, y)	$(-1, -10)$
intervals $[a, b]$ *where increasing*	$[-1, 3]$
intervals $[a, b]$ *where decreasing*	$[-\infty, -1]$, $[3, \infty]$
inflection points (x, y)	$(1, 6)$
intervals $[a, b]$ *where concave up*	$[-\infty, 1]$
intervals $[a, b]$ *where concave down*	$[1, \infty]$
y-intercept	$y = -5$

Take the derivative of $f(x)$ to get $f'(x) = -3x^2 + 6x + 9 = -3(x^2 - 2x - 3) = -3(x+1)(x-3)$. From the factorization, it is clear that $f'(x) = 0$ when $x = -1,3$, so these are critical points of $f(x)$. Plug in an x from each of the intervals $x < -1$, $-1 < x < 3$, and $x > 3$ to determine whether $f(x)$ is increasing or decreasing on these intervals. For instance, $f'(-2) = -15 < 0$, $f'(0) = 9 > 0$, and $f'(4) = -15 < 0$, so $f(x)$ is decreasing on the intervals $(-\infty, -1)$ and $(3, \infty)$ and increasing on the interval $(-1, 3)$. This also means that $x = -1$ is a local minimum and $x = 3$ is a local maximum.

To find out about the concavity of $f(x)$, we look at $f''(x) = -6x + 6$. Clearly, $f''(x) = 0$ only for $x = 1$, so this is a potential inflection point. We check the sign of the second derivative on either side of $x = 1$ to see if the concavity is changing. For example, $f''(0) = 6 > 0$ so f is concave up to the left of 1, and $f''(2) = -6 < 0$, so f is concave down to the right of 1. Thus, 1 is an inflection point.

Now to sketch the graph, we need to know where to place it on the plane, so we find the values of our maximum and minimum, as well as our y-intercept. We get $f(-1) = 1 + 3 - 9 - 5 = -10$, $f(3) = -27 + 27 + 27 - 5 = 22$, and $f(0) = -5$. Now sketch a graph with all this information. It should look like a negative cubic.



Part b.) <u>On the closed interval</u> $[-2, 5]$, what is the global maximum of $f(x)$, and at what point (or points) is this maximum value attained? Answer the same question for the global minimum on this interval.

Answer: To find the global max and min on this interval, we need to find the value of $f(x)$ *at all critical points and endpoints, since these are the only points where extrema can occur. We already know* $f(-1) = -10$ *and* $f(3) = 22$. *Additionally, we can find* $f(-2) = 8 + 12 - 18 - 5 = -3$, *and* $f(5) = -125 + 75 + 45 - 5 = -10$. *Thus, our maximum value is* 22 *and occurs at* $(3, 22)$ *and our minimum value is* -10 *and is achieved at* $(-1, -10)$ *and* $(5, -10)$.

Question 4. (18 points)
Part a.) Sketch the graph of $y = g(z) = z^3 - 6z^2 + 9z - 7$. You do not need to find the z-intercepts, but all other information about the graph should be filled in the following table.

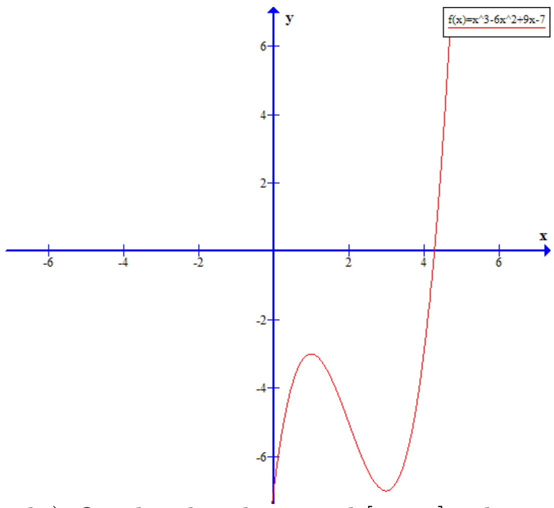
Answer:

critical points (z, y)	$(1, -3)$, $(3, -7)$
relative maxima (z, y)	$(1, -3)$
relative minima (z, y)	$(3, -7)$
intervals $[a, b]$ *where increasing*	$[-\infty, 1]$, $[3, \infty]$
intervals $[a, b]$ *where decreasing*	$[1, 3]$
inflection points (z, y)	$(2, -5)$
intervals $[a, b]$ *where concave up*	$[2, \infty]$
intervals $[a, b]$ *where concave down*	$[-\infty, 2]$
y-intercept	$y = -7$

Take the derivative of $g(z)$ *to get* $g'(z) = 3z^2 - 12z + 9 = 3(z^2 - 4z + 3) = 3(z-3)(z-1)$. *From the factorization, it is clear that* $g'(z) = 0$ *when* $z = 1{,}3$, *so these are critical points of* $g(z)$. *Plug in a* z *from each of the intervals* $z < 1$, $1 < z < 3$, *and* $z > 3$ *to determine whether* $g(z)$ *is increasing or decreasing on these intervals. For instance,* $g'(0) = 9 > 0$, $g'(2) = -3 < 0$, *and* $g'(4) = 3 > 0$, *so* $g(z)$ *is increasing on the intervals* $(-\infty, 1)$ *and* $(3, \infty)$ *and decreasing on the interval* $(1, 3)$. *This also means that* $z = 1$ *is a local maximum and* $z = 3$ *is a local minimum.*

To find out about the concavity of $g(z)$, *we look at* $g''(z) = 6z - 12$. *Clearly,* $g''(z) = 0$ *only for* $z = 2$, *so this is a potential inflection point. We check the sign of the second derivative on either side of* $z = 2$ *to see if the concavity is changing. For example,* $g''(0) = -12 < 0$ *so* g *is concave down to the left of* 2, *and* $g''(3) = 6 > 0$, *so* g *is concave up to the right of* 2. *Thus,* 2 *is an inflection point.*

Now to sketch the graph, we need to know where to place it on the plane, so we find the values of our maximum and minimum, as well as our y-intercept. We get $g(1) = 1 - 6 + 9 - 7 = -3$, $g(3) = 27 - 54 + 27 - 7 = -7$, *and* $g(0) = -7$. *Now sketch a graph with all this information. It should look like a cubic.*



Part b.) On the closed interval $[-2, 4]$, what is the global maximum of $g(z)$, and at what point (or points) is this maximum value attained? Answer the same question for the global minimum on this interval.

Answer: *To find the global max and min on this interval, we need to find the value of* $g(z)$ *at all critical points and endpoints, since these are the only points where extrema can occur. We already know* $g(1) = -3$ *and* $g(3) = -7$*. Additionally, we can find* $g(-2) = -8 - 24 - 18 - 7 = -57$*, and* $g(4) = 64 - 96 + 36 - 7 = -3$*. Thus, our maximum value is* -3 *and occurs at* $(1, -3)$ *and* $(4, -3)$ *and our minimum value is* -57 *and is achieved at* $(-2, -57)$*.*

Question 4. (18 points)
Part a.) Sketch the graph of $y = h(t) = -2t^3 - 3t^2 + 12t + 11$. You do not need to find the t-intercepts, but all other information about the graph should be filled in the following table:

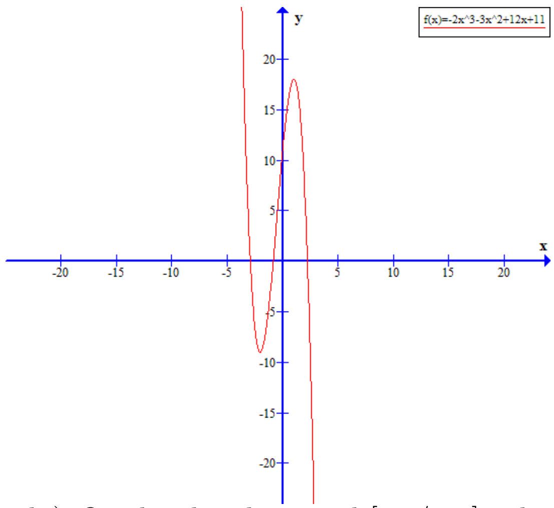
Answer:

critical points (t, y)	$(-2, -9)$, $(1, 18)$
relative maxima (t, y)	$(1, 18)$
relative minima (t, y)	$(-2, -9)$
intervals $[a, b]$ *where increasing*	$[-2, 1]$
intervals $[a, b]$ *where decreasing*	$[-\infty, -2]$, $[1, \infty]$
inflection points (t, y)	$(-\frac{1}{2}, \frac{9}{2})$
intervals $[a, b]$ *where concave up*	$[-\infty, -\frac{1}{2}]$
intervals $[a, b]$ *where concave down*	$[-\frac{1}{2}, \infty]$
y-intercept	$y = 11$

Take the derivative of $h(t)$ *to get* $h'(t) = -6t^2 - 6t + 12 = -6(t^2 + t - 2) = -6(t + 2)(t - 1)$. *From the factorization, it is clear that* $h'(t) = 0$ *when* $t = -2,1$*, so these are critical points of* $h(t)$. *Plug in a* t *from each of the intervals* $t < -2$, $-2 < t < 1$, *and* $t > 1$ *to determine whether* $h(t)$ *is increasing or decreasing on these intervals. For instance,* $h'(-3) = -24 < 0$, $h'(0) = 12 > 0$, *and* $h'(2) = -24 < 0$, *so* $h(t)$ *is decreasing on the intervals* $(-\infty, -2)$ *and* $(1, \infty)$ *and increasing on the interval* $(-2, 1)$. *This also means that* $t = -2$ *is a local minimum and* $t = 1$ *is a local maximum.*

To find out about the concavity of $h(t)$*, we look at* $h''(t) = -12t - 6$. *Clearly,* $h''(t) = 0$ *only for* $t = -1/2$*, so this is a potential inflection point. We check the sign of the second derivative on either side of* $t = -1/2$ *to see if the concavity is changing. For example,* $h''(-1) = 6 > 0$ *so* h *is concave up to the left of* $-1/2$*, and* $h''(0) = -6 < 0$*, so* h *is concave down to the right of* $-1/2$. *Thus,* $-1/2$ *is an inflection point.*

Now to sketch the graph, we need to know where to place it on the plane, so we find the values of our maximum and minimum, as well as our y*-intercept. We get* $h(-2) = 16 - 12 - 24 + 11 = -9$, $h(1) = -2 - 3 + 12 + 11 = 18$*, and* $h(0) = 11$. *Now sketch a graph with all this information. It should look like a negative cubic.*



Part b.) <u>On the closed interval</u> $[-7/2, 2]$, what is the global maximum of $h(t)$, and at what point (or points) is this maximum value attained? Answer the same question for the global minimum on this interval.

Answer: To find the global max and min on this interval, we need to find the value of $h(t)$ *at all critical points and endpoints, since these are the only points where extrema can occur. We already know* $h(-2) = -9$ *and* $h(1) = 18$*. Additionally, we can find* $h(-7/2) = -2(-7/2)^3 - 3(7/2)^2 + 12(7/2) + 11 = 18$*, and* $h(2) = -16 - 12 + 24 + 11 = 7$*. Thus, our maximum value is* 18 *and occurs at* $(1, 18)$ *and* $(-7/2, 18)$ *and our minimum value is* -9 *and is achieved at* $(-2, -9)$*.*

Question 5. (15 points) There is a cylindrical can of radius x and height y. Given that the can's area must be 12π, what should x and y be so that the volume is maximized?

Answer: The area can be expressed as

$$A = 2\pi xy + 2\pi x^2$$

and the volume is

$$V = \pi x^2 y$$

Setting $A = 12\pi$ *and solving for* y *we get that*

$$y = \frac{6}{x} - x$$

Substituting this into the equation for V *we get a function solely in terms of* x*:*

$$V = \pi x^2 \left(\frac{6}{x} - x \right) = 6\pi x - \pi x^3$$

Taking the derivative and setting it equal to zero, we have

$$\frac{dV}{dx} = 6\pi - 3\pi x^2 = 0 \Rightarrow x = \sqrt{2}$$

Now substituting this into the equation for y*, we have*

$$y = 2\sqrt{2}$$

Question 5. (15 points) There is a cylindrical can of radius s and height l. Given that the can's area must be 18π, what should s and l be so that the volume is maximized?
Answer: The area can be expressed as

$$A = 2\pi sl + 2\pi s^2$$

and the volume is

$$V = \pi s^2 l$$

Setting $A = 18\pi$ *and solving for* l *we get that*

$$l = \frac{9}{s} - s$$

Substituting this into the equation for V *we get a function solely in terms of* s*:*

$$V = \pi s^2 \left(\frac{9}{s} - s \right) = 9\pi s - \pi s^3$$

Taking the derivative and setting it equal to zero, we have

$$\frac{dV}{ds} = 9\pi - 3\pi s^2 = 0 \Rightarrow s = \sqrt{3}$$

Now substituting this into the equation for l*, we have*

$$l = 2\sqrt{3}$$

Question 5. (15 points) There is a cylindrical can of radius t and height m. Given that the can's area must be 30π, what should t and m be so that the volume is maximized?
Answer: *The area can be expressed as*

$$A = 2\pi tm + 2\pi t^2$$

and the volume is

$$V = \pi t^2 m$$

Setting $A = 30\pi$ *and solving for* m *we get that*

$$m = \frac{15}{t} - t$$

Substituting this into the equation for V *we get a function solely in terms of* t*:*

$$V = \pi t^2 \left(\frac{15}{t} - t \right) = 15\pi t - \pi t^3$$

Taking the derivative and setting it equal to zero, we have

$$\frac{dV}{dt} = 15\pi - 3\pi t^2 = 0 \Rightarrow t = \sqrt{5}$$

Now substituting this into the equation for m*, we have*

$$m = 2\sqrt{5}$$