University of California at Berkeley
Electrical Engineering and Computer Science
EE105 Midterm Examination #1
Feb. 25, 2016
(80 minutes)

CLOSED BOOK: One standard 8.5” x 11” sheet of notes (both sides) permitted

- Read each problem completely and thoroughly before beginning to work on it
- Summarize all your answers in the boxes provided on these exam sheets
- Show your work in the space provided so we can check your work and scan for partial credit
- Remember to put your name in the space above

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1. (30 points) Frequency response. Assume an ideal opamp.

(a) (10 points) Derive the small-signal transfer function. \( H(j\omega) = \frac{V_o(j\omega)}{V_i(j\omega)} \).

\[ H(j\omega) = \frac{\frac{R}{1+2\omega(100RC)}}{\frac{1}{1+2\omega(100RC)}} \]

\[ H(j\omega) = -100 \left( \frac{1+2\omega(100RC)}{1+2\omega(100RC)} \right) = -100 \quad (\text{pole-zero cancel}) \]

(b) (10 points) Determine the pole(s) and zero(s) frequencies. (Note: Be careful with \( \omega \) versus \( f \))

Zero freq: \( \omega_z(100RC) = 1 \Rightarrow \omega_z = \frac{1}{100RC} \text{ rad/sec} \)

Pole freq: \( \omega_p(100RC) = 1 \Rightarrow \omega_p = \frac{1}{100RC} \text{ rad/sec} \)

Pole-zero cancellation \( \Rightarrow H(j\omega) = -100 \ (40 \text{ dB}) \)

(c) (10 points) Sketch the Bode magnitude and phase plots (label all constant gain and phase levels and all breakpoint frequencies).

\[ 1H(j\omega) = 100 \ (40 \text{ dB}) \]

\[ \angle H(j\omega) = -180^\circ \ (\text{corresponding to } \cdot) \]

This is an \textit{upass} filter (e.g., oscilloscope probe input)
2. (35 points). CMOS bias calculations. For the NMOS, \( V_{in} = 0.7 \text{ V} \), \( k_n = 1.5 \text{ mA/V}^2 \), and \( \lambda_n = 0 \). For the PMOS, \( V_{ip} = -0.7 \text{ V} \), \( k_p = 0.5 \text{ mA/V}^2 \), and \( \lambda_p = 0 \).

Find the values of \( I \), \( V_1 \), \( V_2 \), and \( V_3 \).

- Overall, 4 equations in 4 unknowns:
  \[ I = \frac{V_1}{R_1} = \frac{V_1}{1k} \quad \ldots \quad (1) \]
  \[ I = \frac{k_n}{2} (V_{gs1} - V_{th})^2 \]
  \[ I = \frac{k_p}{2} (V_{gs2} - V_{th})^2 = \frac{k_n}{2} (V_2 - V_1 - 0.7)^2 \quad \ldots \quad (2) \]
  \[ = \frac{k_p}{2} (V_3 - V_2 - 0.7)^2 \quad \ldots \quad (3) \]
  \[ I = \frac{V_{dd} - V_3}{R_2} \quad \ldots \quad (4) \]

- Now, consider (2) and (3):
  \[ \frac{V_{dd} - V_3}{1k} = \frac{V_1}{1k} - \frac{V_3}{1k} \]

\[ I = \frac{k_n}{2} (V_{gs1} - 0.7)^2 = \frac{k_p}{2} (V_{gs2} - 0.7)^2 \Rightarrow V_{gs2} = 1.73 V_{gs1} - 0.51 \quad (5) \]

\[ KV_L \Rightarrow \quad IR_1 + V_{gs1} + V_{gs2} + IR_2 = V_{dd} \]

Thus, \[ I = \frac{\frac{V_{dd} - V_{gs1} - V_{gs2}}{2}}{2k} \]

Equate (6) to (2):

\[ 10^{-3} (2.76 - 1.37 V_{gs1}) = 0.75 \times 10^{-3} (V_{gs1} - 0.7)^2 \]

\[ 1.33 (2.76 - 1.37 V_{gs1}) = V_{gs1}^2 - 1.4 V_{gs1} + 1.49 \]

\[ \Rightarrow V_{gs1}^2 + 0.42 V_{gs1} - 3.18 = 0 \]

\[ \Rightarrow V_{gs1} = -0.42 \pm 3.59 = \frac{1.59 V \text{ or } -2.01 V}{2} \]

\[ V_{gs2} = 1.73 V_{gs1} - 0.51 = 2.24 V \]

\[ I = \frac{\frac{V_{dd} - V_{gs1} - V_{gs2}}{2}}{2k} = 0.59 \text{ mA} \]

\[ I = \frac{V_1}{R_1} = 0.59 V \quad \Rightarrow \quad V_1 = 0.59 V \]

\[ V_2 = V_3 - V_{gs2} = 2.17 V \]

\[ V_3 = 4.41 V \]
3. **(35 points)** Assuming an ideal opamp, design the current amplifier shown below to have a current gain of $I_L/I_i = 11$ A/A.

(a) Find the required value for $R$.

Note: $\frac{I_L}{I_i} = 11 \Rightarrow I_L = 11I_i$.

KCL @ In: $I_i + I_R = I_L = 11I_i$.

\[ I_R = 10I_i = 0 - \frac{V_x}{R} \]

But $V_x = 0 - I_i \cdot (10k) = -(10k)I_i$.

\[ 10RI_i = -(10k \cdot I_i) \Rightarrow 10R = 10k \]

\[ R = \frac{10k}{10} = 1k \]

\[ R = 1k \Omega \]

(b) What is the input resistance of this current amplifier, $R_{in}$?

Use test source to find $R_{in}$.

\[ R_{in} = \frac{V_{TEST}}{I_{TEST}} = \frac{0}{I_{TEST}} = 0 \]

\[ R_{in} = 0 \Omega \]
(c) If \( R_L = 1 \, \text{k}\Omega \) and the opamp operates in an ideal manner as long as \( V_o \) is in the range \(+12 \, \text{V}\) to \(-12 \, \text{V}\), what range of \( I_i \) is possible?

\[
V_o = 0 - I_i (10k) - 11I_i (1k)
\]
\[
= -10k I_i - 11k I_i
\]
\[
= -21k I_i
\]

\[
I_i^{(\text{max})} = \frac{V_o^{(\text{max})}}{21k} = \frac{-12 \, \text{V}}{21k} = -0.57 \, \text{mA}
\]

\[
I_i^{(\text{min})} = \frac{-V_o^{(\text{min})}}{21k} = \frac{-(-12 \, \text{V})}{21k} = +0.57 \, \text{mA}
\]

\[
I_i \text{ range} = -0.57 \, \text{mA} \leq I_i \leq 0.57 \, \text{mA}
\]

(d) If the amplifier is fed with a current source having a current of 0.2 mA and a source resistance of 10 k\(\Omega\), find \( I_L \).

Because of ideal opamp with negative feedback,

\( V_{in^+} = V_{in^-} = 0 \, \text{V} \).

\( \circ \) Current through \( R_S \), \( I_R = 0 \).

\( \circ \) \( I_i = I_S = 0.2 \, \text{mA} \).

\( \circ \) \( I_L = 11I_i = 2.20 \, \text{mA} \).