Prof. Battaglia Spring 04 Mid term 1.

Problem 1.

For derivation, please see pp. 795-797.

Points were given for showing the following:

① Proof starting with Maxwell's Eqns in vacuum. (Eqns correctly written).

② Achieving $\frac{\partial B}{\partial x} = -\mu_0 \varepsilon_0 \frac{\partial E}{\partial t}$ by doing the loop integral.

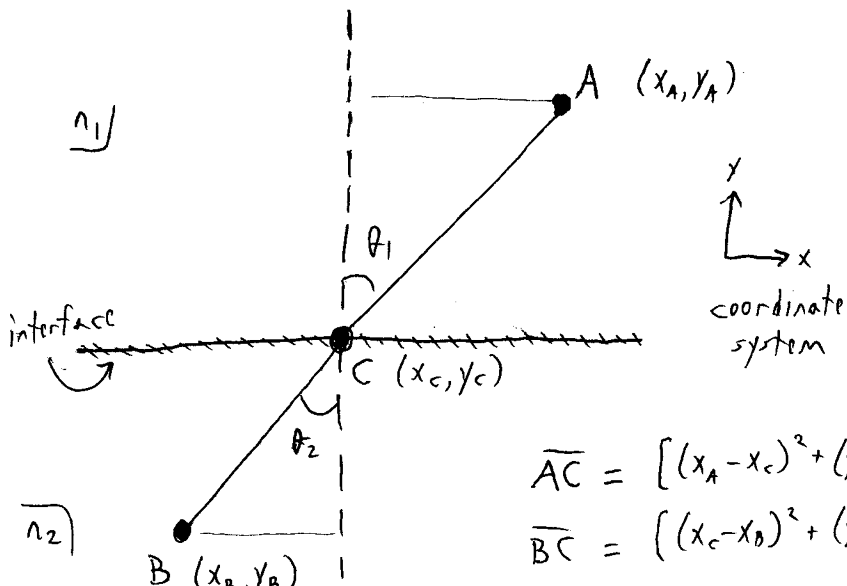
③ Achieving $\frac{\partial E}{\partial x} = -\frac{\partial B}{\partial t}$ by doing the other loop integral.

④ Equating $\frac{\partial^2 B}{\partial t \partial x} = \frac{\partial^2 B}{\partial x \partial t}$.

⑤ Achieving wave equation $\frac{\partial^2 E}{\partial t^2} = \frac{1}{\mu_0 \varepsilon_0} \frac{\partial^2 E}{\partial x^2}$.

⑥ Finding from ⑤ that $c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$.

7C Spring '04
Midterm #1, Battaglia
Problem #2



technicality: there's also a z coordinate. I've implicitly defined A and B to live in the xy-plane and assumed that C does also. this is fine, since moving C in z will only increase the light's transit time. I didn't grade on this.

$$\overline{AC} = \left[(x_A - x_c)^2 + (y_A - y_c)^2\right]^{1/2}$$
$$\overline{BC} = \left[(x_c - x_B)^2 + (y_c - y_B)^2\right]^{1/2}$$

$$\sin\theta_1 = (x_A - x_c)/\overline{AC}$$
$$\sin\theta_2 = (x_c - x_B)/\overline{BC}$$

note that $x_A > x_c > x_B$ yields $\sin\theta_1$ and $\sin\theta_2$ both positive, which is what we want

10 pts a) find time for light transit from A to B through C

material # 1: $t_1 = \dfrac{\overline{AC}}{v_1} = \dfrac{n_1 \overline{AC}}{c}$

material # 2: $t_2 = \dfrac{\overline{CB}}{v_2} = \dfrac{n_2 \overline{CB}}{c}$

total: $t_{tot} = t_1 + t_2 = \dfrac{n_1 \overline{AC}}{c} + \dfrac{n_2 \overline{CB}}{c}$

15 pts b) use Fermat principle of least time to prove Snell's Law

in coordinates,

$$t_{tot} = \frac{n_1}{c}\left((x_A - x_c)^2 + (y_A - y_c)^2\right)^{1/2} + \frac{n_2}{c}\left[(x_c - x_B)^2 + (y_c - y_B)^2\right]^{1/2}$$

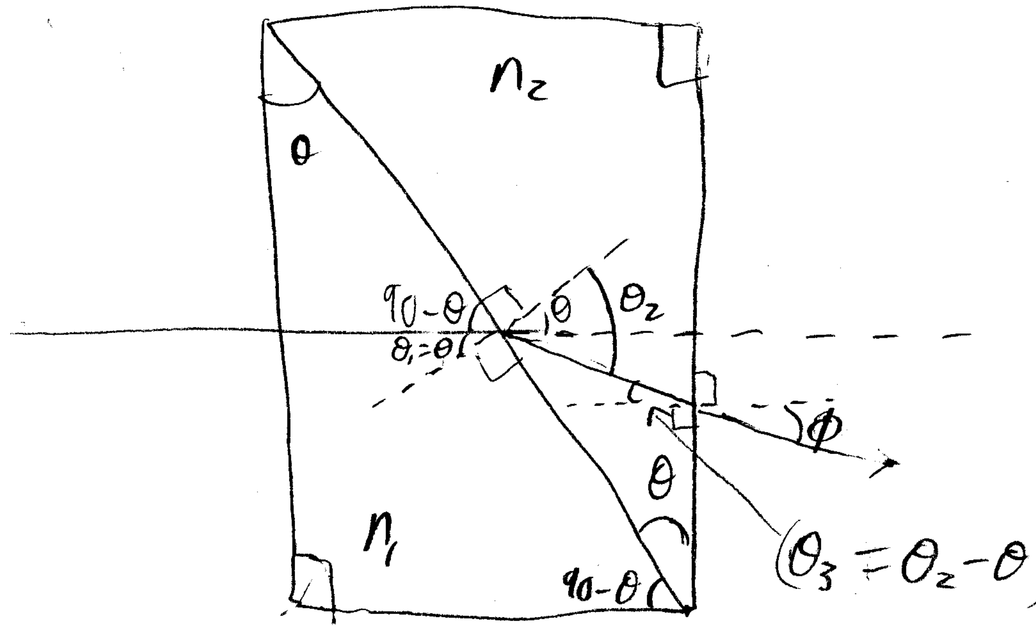
minimize transit time w.r.t. x_c (y_c is fixed at interface)

$$\frac{dt_{tot}}{dx_c} = \frac{n_1}{c}\frac{1}{2}\frac{2(x_A - x_c)(-1)}{\overline{AC}} + \frac{n_2}{c}\frac{1}{2}\frac{2(x_c - x_B)}{\overline{BC}}$$

$$0 = \frac{n_1}{c}(-\sin\theta_1) + \frac{n_2}{c}\sin\theta_2$$

$$\Rightarrow \quad n_1 \sin\theta_1 = n_2 \sin\theta_2$$

[3]



(1) $n_1 \sin\theta_1 = n_2 \sin\theta_2 \Rightarrow \theta_2 = \sin^{-1}\left(\frac{n_1}{n_2}\sin\theta\right)$

(2) $n_2 \sin\theta_3 = n_{air}\sin\phi \Rightarrow \theta_2 = \sin^{-1}\left(\frac{1}{n_2}\sin\phi\right)+\theta$

USING $\sin(A+B) = \sin A\cos B + \cos A \sin B$:

$$\frac{n_1}{n_2}\sin\theta = \frac{1}{n_2}\sin\phi\cos\theta + \sqrt{1-\frac{1}{n_2^2}\sin^2\phi}\,\sin\theta$$

$$n_1\tan\theta = \sin\phi + \sqrt{n_2^2-\sin^2\phi}\,\tan\theta \leftarrow \text{ICK!}$$

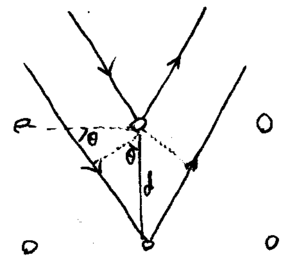
FOR $\theta, \phi \ll 1$; $n_1, n_2 \sim 1$:

$$\rightarrow n_1\theta \simeq \phi + n_2\theta \Rightarrow \boxed{\Delta n = n_1 - n_2 \simeq \frac{\phi}{\theta}}$$

Bataglia

Cerke

4.



a). $2d \sin\theta = m\lambda$

b) $\sin\theta_m = \frac{m\lambda}{2d}$

$\sin\theta_0 = 0 \Rightarrow \theta_0 = 0$

$\sin\theta_1 = 0.716 = \frac{\lambda}{2d}$. $\theta_1 \approx 46°$

$\sin\theta_2 > 1$.

c). The analysis can't be performed by using visible light, because the minimum value of the wavelength of visible light is far more greater than the spacing between two ions, if we use the formula $\sin\theta = \frac{m\lambda}{2d}$, we can see as long as $m > 0$, $\sin\theta$ will exceed 1, which is not acceptable.

5.

$\theta = \dfrac{1.22\lambda}{D}$. $D = 2.4m$, $d = 1300$ light year

$\Delta l = \theta d = \dfrac{1.22\lambda}{D} \times 1300\ ly$

if we choose $\lambda = 115\ nm$, we get $\Delta l = 7.6 \times 10^{-5}\ ly$

if we choose $\lambda = 2500\ nm$, we get $\Delta l = 1.65 \times 10^{-3}\ ly$