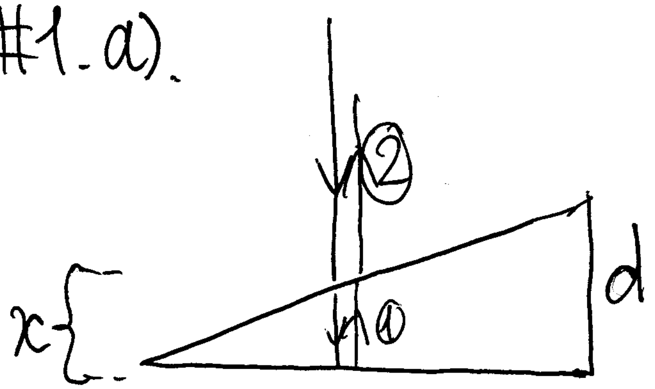
Lee.

#1. a).



$$\Delta\phi = \overbrace{K\cdot 2x}^{\text{extra distance traveled}} + \underbrace{\pi}_{\text{on reflection}}$$

(+5) ~~(+10)~~

+5

$\Delta\phi = 2\pi m + \pi$ (+5)

$m = 0, 1, 2 \ldots$

$$2\pi\cdot 6 + \pi = \frac{2\pi}{\lambda}\cdot 2d + \cancel{\pi}$$ (+10)

−5 if m=7

−2 for bad math / no math

$\frac{d}{\lambda} = 3 \Rightarrow \boxed{d = 1800\ \text{nm}}$

b). (+5) no phase-shift on reflection.

(+5) $\lambda \rightarrow \frac{\lambda}{n}$

$$2\pi\cdot m + \pi \leq 2\pi\cdot 2\frac{d}{\lambda}n$$ (+5)

$$m \leq 6\cdot 1.33 - \frac{1}{2} = \frac{15}{2} \Rightarrow m = \cancel{8}\,7$$

$\boxed{\text{8 fringes}}$

Lee #2

2a). $n_g \sin \theta_c = n_\ell \sin 90$ +30

$$\boxed{\frac{n_\ell}{n_g} = \sin \theta_c}$$

b). $n_\ell = 1$

$n_g = \frac{1}{\sin 45} = \sqrt{2} = 1.41$ +5

$\sin 58 = \frac{\cancel{\sqrt{2}}\, n_\ell}{\sqrt{2}}$

$n_\ell = \sqrt{2} \sin 58 = 1.19$ +5

3) a) Total energy $E = \sqrt{p^2c^2 + m^2c^4}$

Kinetic energy = total energy − rest energy

$$E_k = \sqrt{p^2c^2 + m^2c^4} - mc^2$$

$$\Rightarrow E_k + mc^2 = \sqrt{p^2c^2 + m^2c^4}$$

$$E_k^2 + \cancel{m^2c^4} + 2E_k mc^2 = p^2c^2 + \cancel{m^2c^4}$$

$$p^2c^2 = E_k^2 + 2E_k mc^2$$

$$= (1\,\text{keV})^2 + 2(1\,\text{keV})(511\,\text{keV}/c^2)c^2$$

$$= 1\,\text{keV}^2 + 1022\,\text{keV}^2$$

$$p^2c^2 = 1023\,\text{keV}^2$$

$$p = \sqrt{1023\,\frac{\text{keV}^2}{c^2}} = \boxed{32.0\,\text{keV}/c = p}$$

$$(= 1.07\times10^{-7}\,\text{keV}\cdot\text{s/m})$$
$$(= 1.71\times10^{-23}\,\text{kg m/s})$$

−8 using nonrel w/o justifying $E_k = \frac{1}{2}mv^2 = \frac{p^2}{2m}$

−4 wrong answer, major mistake

−2 wrong answer, math error

b) electrons → waves of $\lambda = \lambda_{\text{deBroglie}} = h/p$

1^{st} min of diffraction = Same equation as 1^{st} maximum of interference

$$a \sin\theta_{min} = \lambda = \lambda_{\text{deBroglie}} = h/p$$

$$a = \frac{h}{p\sin\theta_{min}}$$

$\theta_{min} = 10° \times \frac{\pi\ \text{radians}}{180°} = 0.17$ radians $\ll 1 \Rightarrow$ Small angle approx

$$a = \frac{h}{p\theta_{min}} = \frac{hc}{pc\,\theta_{min}} = \frac{1240\,\text{eV}\cdot\text{nm}}{(32.0\times10^3\,\text{eV})(0.17)} = \boxed{0.22\,\text{nm} = a}$$

2.2 Å ≃ size of atoms − small!

a crystal lattice could be used for this

−2 $\frac{a}{2}\sin\theta_{min} = \lambda$

−4 $p = mv$ w/o justification

−2 minor math error

−4 if it causes a ridiculous answer & you don't acknowledge it

−2 no or wildly incorrect slit origin

c) $E_k = (\gamma - 1)\, mc^2$

$$1\,\text{keV} = (\gamma - 1)\left(511\ \text{keV}/c^2\right)c^2$$

$$\frac{1\,\text{keV}}{511\,\text{keV}} = \gamma - 1$$

$$\Rightarrow \gamma = 1 + \frac{1}{511}$$

Thus γ differs from 1 by $100 \times \frac{1}{511}\% = \frac{100}{511}\% \sim \frac{100}{500} \sim .2\%$

Thus it is $\underline{\text{not}}$ relativistic.

+5 $\gamma = 1 + 1/511$ or something like that

or

$v = 0.06c$

+5 "nonrel" (or "rel" if incorrect math above gives $\gamma \gg 1$, etc)

+5 $\gamma = 1 + 20\%$

or $\gamma \simeq 1$ or something like that

Lee #4:

$$\gamma = \frac{1}{\sqrt{1-\frac{3}{4}}} = 2$$

a). $\Delta L = \Delta L_0 / \gamma = L/2$ (+4)

similarly

$$L_{barn} = \frac{L_0/2}{\gamma} = \frac{L_0}{4}$$ (+4)

Yes (+2)

b). No. (+3)

$$(\Delta x)^2 - (\Delta t)^2 c^2 = (\Delta x')^2 - (\Delta t')^2 c$$

$$(L)^2 - c^2(\Delta t)^2 = \left(\frac{L}{2}\right)^2$$

$$c^2(\Delta t)^2 = \frac{3}{4} L^2$$

$$\boxed{\Delta t = \frac{\sqrt{3}}{2} L/c}$$ (+10)

b). It tells you that the far away door closes first. +2

c). $L_{barn} = \frac{L}{4}$

$V = \frac{\sqrt{3}}{2} c$

$\Downarrow$

$\frac{3}{4} L = \frac{\sqrt{3}}{2} c \cdot t_{min}$

$$\boxed{t_{min} = \frac{\sqrt{3}}{2} \frac{L}{c}}$$

+10

No. +5

5) a) Electrons - fermions - no two electrons can be in the same state.

Spin ½ - two electron's can each have the state $\Psi_{n_x n_y}$, the spin up state and the spin down state.

note the limits on n_x & n_y → $n_i = 1, 2, 3, \ldots$

$E_{n_x n_y} =$ $E_{11} = E_0(1^2+1^2) = 2E_0$ — ground state for one electron

$E_{12} = E_0(1+4) = 5E_0$ } 1st excited states

$E_{21} =$ " $= 5E_0$

$E_{22} = E_0(4+4) = 8E_0$ — 2nd excited state

$E_{31} = E_0(9+1) = 10E_0$ } 3rd excited state

$E_{13} =$ " $= 10E_0$

$$E = 2E_{11} + 2E_{12} + 2E_{21} + 1E_{22}$$

two electrons/state

Seventh electron is last electron

$$= E_0(2\cdot2 + 5\cdot2 + 5\cdot2 + 8)$$

$$= E_0(4 + 10 + 10 + 8) = \boxed{32E_0 = E_{gs}}$$

note: if you forgot spin

$E = E_{11} + E_{12} + E_{21} + E_{22} + E_{31} + E_{13} + E_{23}$

$= E_0(2 + 2\cdot5 + 8 + 2\cdot10 + 13)$

$= 53\ E_0$

+5 spin (2 states/energy)

+10 putting electrons in increasing energy orbits

+5 correct order ; $E_{11} + E_{12} + E_{21} + E_{22}$

+1 → +5 correct answer or showing something worthy if you didn't do much

+2 only for doing something w/ hydrogenic (l, m_l) functions

b) Without the pauli exclusion principle, all states can go to the g.s.

$$E = 7 \cdot E_{11} = 7 \cdot 2E_0 = \boxed{14 E_0 = E_{gs}}$$

-3 for math error

If you assumed pauli exclusion still holds, just that there was one electron per $n_x n_y$ then we need the 1st 7 energies

What is the state after $E_{31} = E_{13}$?

$E_{41} = 17 E_0$

$E_{33} = 18 E_0$

$E_{32} = 13 E_0 \leftarrow$

-7 for doing this

-3 for doing it incorrectly

$$\Rightarrow E = E_{11} + E_{21} + E_{21} + E_{22} + E_{31} + E_{23} + E_{32}$$

$$= E_0 (2 + 2 \cdot 5 + 8 + 2 \cdot 10 + 13)$$

$$= E_0 (2 + 10 + 8 + 20 + 13) = 61 E_0$$

6) a) Outside box: $\psi(x)=0$

inside box

$$-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi + 0\cdot\psi = E\psi$$

+2 $\psi = A\sin kx + B\cos kx$, $k=\sqrt{\frac{2mE}{\hbar^2}}$

BCs: $\psi(\frac{L}{2})=0$ +2

$\psi(-\frac{L}{2})=0$

$\psi(\frac{L}{2}) \overset{(1)}{=} A\sin\frac{kL}{2} + B\cos\frac{kL}{2} = 0$

$\psi(-\frac{L}{2}) = A\sin\frac{-kL}{2} + B\cos\frac{-kL}{2} = 0 = -A\sin\frac{kL}{2} + B\cos\frac{kL}{2}$ (2)

(1)+(2)

$2B\cos\frac{kL}{2}=0$

Either $B=0$

or $\frac{kL}{2} = \frac{(2j+1)\pi}{2}$, $j=0,1,2\ldots$

$k=\frac{n\pi}{L}$, n odd

(1)−(2)

$2A\sin\frac{kL}{2}=0$

Either $A=0$

or

$\sin\frac{kL}{2}=0$

$\Rightarrow \frac{Lk}{2} = j\pi$, $j=1,2$

$k=\frac{2j\pi}{L} = \frac{n\pi}{L}$, n even

+4

thus $\psi = \begin{cases} A_n\sin\frac{n\pi x}{L}, & n \text{ even} \\ B_n\cos\frac{n\pi x}{L}, & n \text{ odd} \end{cases}$ +2

$\psi_1 = A_1 \cos \frac{\pi x}{L}$

$\psi_2 = B_2 \sin \frac{2\pi x}{L}$

$\psi_3 = A_3 \cos \frac{3\pi x}{L}$

$\psi_4 = B_4 \sin \frac{4\pi x}{L}$

b) E_1: $k = \frac{\pi}{L} = \sqrt{\frac{2mE}{\hbar^2}}$

$\Rightarrow \boxed{E_1 = \frac{\pi^2 \hbar^2}{2mL^2}}$

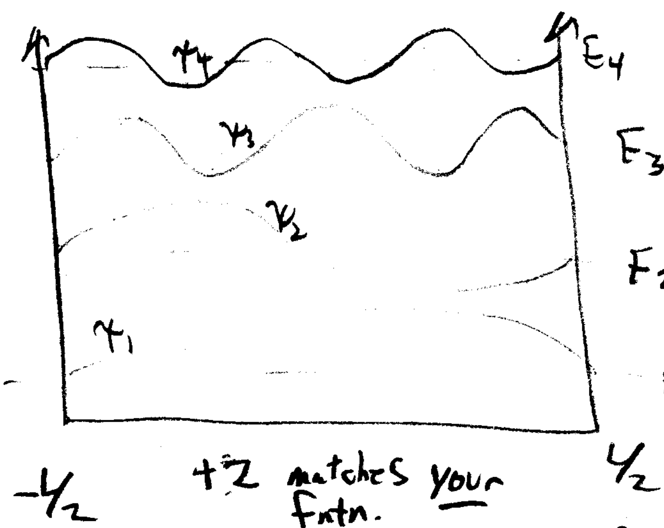
+5 answers
+5 derivation

E_2: $k = \frac{2\pi}{L} = \sqrt{\frac{2mE}{\hbar^2}}$

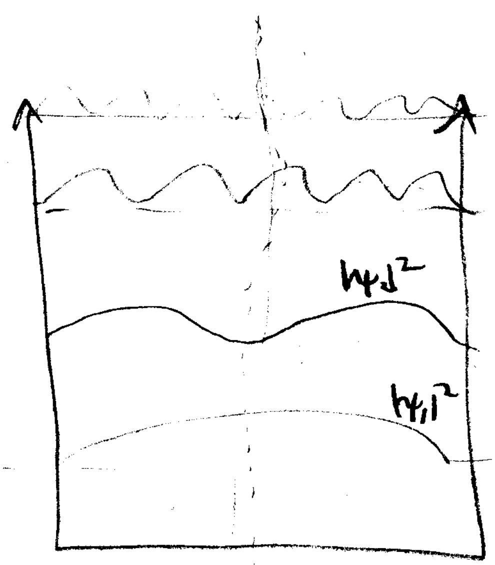
$\Rightarrow \boxed{E_2 = \frac{4\pi^2\hbar^2}{2mL^2}}$

likewise $\boxed{E_3 = 9E_1, \quad E_4 = 16E_1}$

c)



+2 matches your fntn.
+3 matches correct w.f.



+5 is square of ψ

d) $\int_{-\infty}^{\infty} |\psi_n| \, dx = 1$ +5

$|A_n|^2 \int_{-L/2}^{L/2} \sin^2 \frac{n\pi x}{L} dx = 1, \quad n \text{ even}$ +5

$|B_n|^2 \int_{-L/2}^{L/2} \cos^2 \frac{n\pi x}{L} dx = 1, \quad n \text{ odd}$

7) a) $\boxed{iii}$ (+5) b) $\boxed{ii}$ (+5)

c) (i) F (+1)

(ii) T (+1)

(iii) F (+1)

(iv) T (+1)

(v) F (+1)

(no partial credit)