SOLUTION

E-36 SECTION 1

Midterm #2 - November 15, 2006

Problem No1 (60 points)

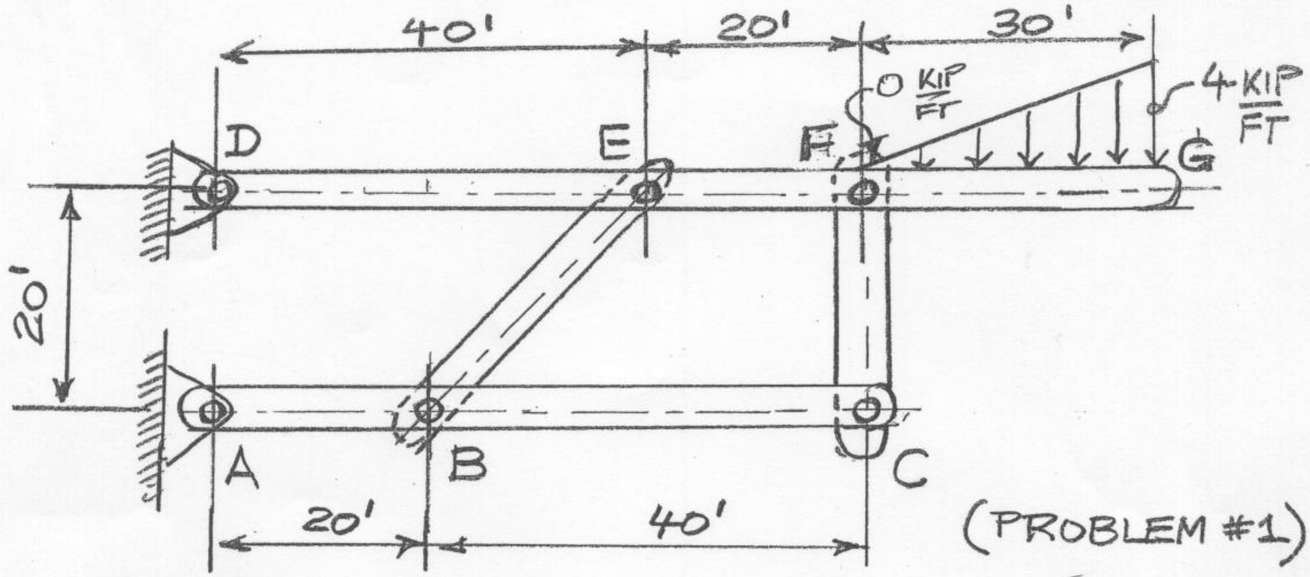
The 2-dimensional figure 1 shows a frame structure with an external hinge support at A and at D respectively.
Both horizontal frame members ABC and DEFG are linked together by means of hinged links BE and CF. Note that that these links are essentially articulated (hinged) at their connection to the horizontal frame members.
A "triangular" distributed load is acting vertically between F and G. (p=0 kip/ft at F and p=4 kip/ft at G).

1. Draw the free body diagram for ABC (5 points)
2. Draw the free body diagram for DEFG (10 points)
3. Draw the free body diagram for link BE (2.5 points)
4. Draw the free body diagram for link CF (2.5 points)
5. Compute all exterior hinge support reactions at A and D (20 points)
6. Compute all link BE and CF internal forces. (20 points)

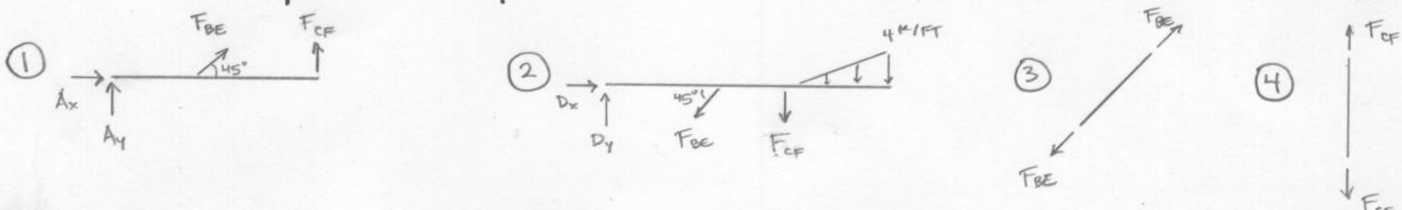
Problem No2 (40 points)

The 2-dimensional figure 2 shows a beam ABCD with external supports at A and B. Support A is a standard hinge and support B is a typical "roller" support providing only vertical restraint.
The loading consists of a "triangular" distributed load acting vertically between A and B. (p=2 kip/ft at A and p=0 kip/ft at B) and of a "force couple" applied at C (two equal and opposite horizontal forces of 60 kip each with a 4 ft. moment arm) .
All dimensions are in feet and are shown on the figure 2.

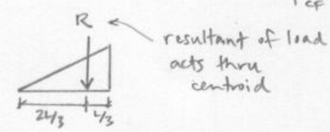
1. Compute all exterior reactions (10 points)
2. Draw the Shear Diagram and give the shear values (15 points)
3. Draw the Moment Diagram and give the moment values (15 points)



(PROBLEM #1)



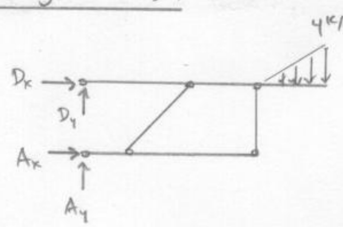
⑤,⑥



from ABC: $\Sigma M_B = 0 = -A_y(20') + F_{CF}(40') \Rightarrow A_y = 2F_{CF}$

from DEFG: $\Sigma M_E = 0 = -D_y(40') - F_{CF}(20') - \frac{1}{2}(4^k/ft)(30')(20' + \frac{2}{3}(30')) \Rightarrow D_y = -0.5F_{CF} - 60^k$

from global FBD:



$\Sigma M_D = 0 = A_x(20') - \frac{1}{2}(4^k/ft)(30')(60' + \frac{2}{3}(30')) \Rightarrow A_x = 240^k$

$\boxed{A_x = 240^k \rightarrow}$

$\Sigma F_x = 0 = D_x + A_x$

$D_x = -A_x = -240^k \Rightarrow \boxed{D_x = 240^k \leftarrow}$

$\Sigma F_y = 0 = A_y + D_y - \frac{1}{2}(4^k/ft)(30')$

substitute from above, $[A_y = 2F_{CF}]$ & $[D_y = -0.5F_{CF} - 60^k]$:

$2F_{CF} + (-0.5F_{CF} - 60^k) - 60 = 0$

$\boxed{F_{CF} = 80^k\text{, tension}}$

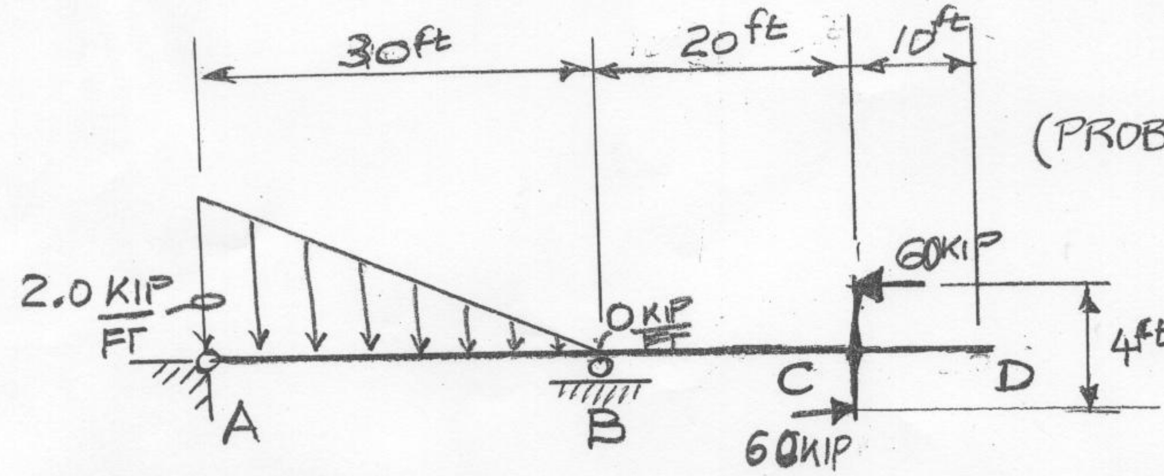
then $A_y = 2F_{CF} = 2(80^k) = 160^k \Rightarrow \boxed{A_y = 160^k \uparrow}$

$D_y = -0.5F_{CF} - 60^k = -0.5(80^k) - 60 \Rightarrow D_y = -100^k$

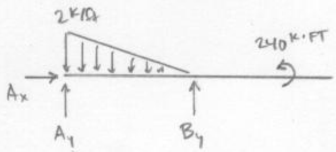
$\boxed{D_y = 100^k \downarrow}$

from ABC: $\Sigma F_x = 0 = A_x + F_{BE}\cos 45° = 240^k + F_{BE}\left(\frac{1}{\sqrt{2}}\right) \Rightarrow F_{BE} = -339^k \Rightarrow \boxed{F_{BE} = 339^k\text{, compression}}$

(PROBLEM #2)



① draw FBD to find reactions:



• remember, resultant of distributed load acts through its centroid; for a triangle this is (1/3)h from the base

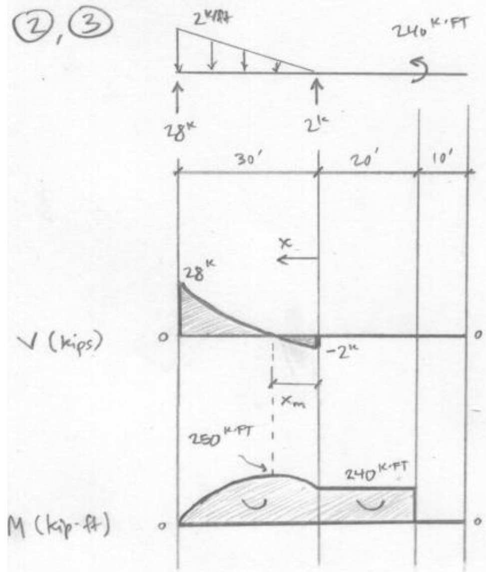
• applied moment at C: $M_C = (60^k)(4') = 240^{k\cdot ft}$

$\Sigma F_x = 0 \Rightarrow \boxed{A_x = 0}$

$\Sigma M_A = 0 = B_y(30') - \frac{1}{2}(2^{k/ft})(30')\left(\frac{1}{3}(30')\right) + 240^{k\cdot ft} \Rightarrow \boxed{B_y = 2^k}$

$\Sigma F_y = 0 = A_y + B_y - \frac{1}{2}(2^{k/ft})(30') = A_y + 2^k - 30^k \Rightarrow \boxed{A_y = 28^k}$

②, ③



shear diagram: use $\frac{dV}{dx} = -w$

• shear at left end = reaction = $\underline{\underline{28^k = V_A}}$

• then shear changes parabolically; using the above equation, as the load w gets smaller, the slope of V gets smaller

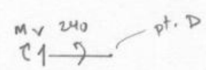
• change in value of the shear diagram is equal to the negative area under the load diagram for a given span

$\therefore \Delta V_{A\to B} = -\frac{1}{2}(30')(2^{k/ft}) = -30^k \longrightarrow \underline{\underline{V_B = -2^k}}$

• shear is zero on the rest of the beam

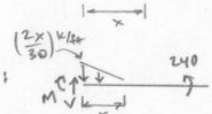
moment diagram: it will be easier to use FBDs rather than the calculus methods that were used for the V diagram:

cut 1:



$\Sigma M_{cut} = 0 = -M + 240^{k\cdot ft} \Rightarrow \underline{\underline{M = 240^{k\cdot ft}}}$ on span BC

cut 2:



note that x-coord axis origin was chosen at B with positive direction to the left, for simplicity:

$\Sigma M_{cut} = 0 = -M + 240^{k\cdot ft} + 2^k(x) - \frac{1}{2}(x)\left(\frac{x}{3}\right)\left(\frac{2x}{30}{}^{k/ft}\right) \Rightarrow M = \left(-\frac{x^3}{90} + 2x + 240\right)^{k\cdot ft}$

M_{max} occurs where $\frac{dM}{dx} = 0$ (from calculus) or also where $V = 0$ since $\frac{dM}{dx} = V$

$\frac{dM}{dx} = -\frac{3x_m^2}{90} + 2 = 0 \Rightarrow x_m = 7.75'$; $M_{max} = -\frac{(7.75)^3}{90} + 2(7.75) + 240 \Rightarrow \underline{\underline{M_{max} = 250^{k\cdot ft}}}$