Name:

SID:

Name of student at your left:
(1 point)
Name of student at your right:
(1 point)

# UNIVERSITY OF CALIFORNIA <br> College of Engineering <br> Department of Electrical Engineering and Computer Sciences 

Midterm 2
EECS 105
B. E. BOSER

October 26, 2004
FALL 2004

- Closed book, closed notes.
- No calculators.
- Copy your answers into marked boxes on exam sheets.
- Simplify numerical and algebraic results as much as possible.

Up to 5 points penalty for results that are not reasonably simplified.

- Mark your name and SID at the top of the exam and all extra sheets.
- Be kind to the graders and write legibly. No credit for illegible results.


## Problem 1 [25 points]



Given: $\quad \mu_{\mathrm{n}} \mathrm{C}_{\mathrm{ox}}=200 \mu \mathrm{~A} / \mathrm{V}^{2}, \mathrm{~V}_{\mathrm{TN}}=1 \mathrm{~V}, \lambda_{\mathrm{n}}=0.01 \mathrm{~V}^{-1} @ \mathrm{~L}=1 \mu \mathrm{~m}$ $\mathrm{g}_{\mathrm{m}} \mathrm{r}_{\mathrm{o}} \gg 1$
The circuit is biased such that all transistors are in saturation.
a) [10 points] Find numerical values (not expressions) for:

$$
\mathrm{I}_{\mathrm{D} 3}=\mathbf{1 0 0 0} \quad \mu \mathrm{A}
$$

$$
\mathrm{g}_{\mathrm{m} 3}=\mathbf{1 0 0 0 0} \quad \mu \mathrm{S}
$$

$$
\begin{aligned}
& I_{D S 3}=I_{D S 2}=\frac{\left(\frac{W}{L}\right)_{2}}{\left(\frac{W}{L}\right)_{1}} \cdot I_{D S 1}=\frac{\left(\frac{100}{1}\right)}{\left(\frac{10}{1}\right)} \cdot 100 \mu A=1000 \mu A \\
& g_{m 3} \approx \sqrt{\mu_{n} C_{o x} \cdot\left(\frac{W}{L}\right)_{3} \cdot 2 \cdot I_{D S 3}}=\sqrt{\left(200 \mu A / V^{2}\right) \cdot\left(\frac{250}{1}\right) \cdot 2 \cdot 1000 \mu A}=10 \mathrm{mS}=10000 \mu \mathrm{~S}
\end{aligned}
$$

b) [15 points] Find an algebraic expression for the small signal output resistance (at terminal $v_{O U T}$ ) of the circuit as a function of transistor small-signal parameters. Use $\mathrm{g}_{\mathrm{m}} \mathrm{r}_{\mathrm{o}}$ $\gg 1$ to simplify your result as much as possible:

$$
\mathrm{r}_{\mathrm{out}}=1 /\left(\mathrm{g}_{\mathrm{m} 3}+\mathrm{g}_{\mathrm{mb} 3}\right)
$$

$$
r_{o u t}=r_{o 2} / / \frac{r_{o 3}}{1+\left(g_{m 3}+g_{m b 3}\right) \cdot r_{o 3}} \approx r_{o 2} / / \frac{1}{\left(g_{m 3}+g_{m b 3}\right)} \approx \frac{1}{\left(g_{m 3}+g_{m b 3}\right)}
$$

## Problem 2 [25 points]



The above sketch shows a rough approximation of the electron drift velocity versus the electrical field in Silicon. For an NMOS transistor with $\mathrm{L}=0.1 \mu \mathrm{~m}, \mathrm{~W}=10 \mu \mathrm{~m}$, and $\mathrm{C}_{\mathrm{ox}}=5 \mathrm{fF} / \mu \mathrm{m}^{2}$ calculate the following:
a) [10 points] What is the minimum $V_{D S}$ (numerical value) for which current flow is limited by the thermal carrier drift velocity? Assume that the field in the channel is uniform.

$$
\mathrm{V}_{\mathrm{DS}}=\mathbf{0 . 1}
$$

$$
E_{\text {critical }}=\frac{1 V}{1 \mu m}=\frac{V_{D S}}{L}=\frac{V_{D S}}{0.1 \mu m} \Rightarrow V_{D S}=0.1 V
$$

b) [15 points] Find the numerical value of the maximum drain current $\mathrm{I}_{\mathrm{D}}$ for $\mathrm{V}_{\mathrm{GS}}-$ $\mathrm{V}_{\mathrm{TH}}=1 \mathrm{~V}$. Hint: get the current from the channel charge and its velocity.

$$
\mathrm{I}_{\mathrm{D}}=4.75 \quad \mathrm{~mA}
$$

Average channel charg $e=\frac{(C h \arg e @ \text { source })+(C h \arg e @ d r a i n)}{2}$
$=\frac{C_{o x} \cdot W \cdot L \cdot\left[\left(V_{G S}-V_{T H}\right)+\left(V_{G D}-V_{T H}\right)\right]}{2}=\frac{C_{o x} \cdot W \cdot L \cdot\left[\left(V_{G S}-V_{T H}\right)+\left(V_{G S}-V_{D S}-V_{T H}\right)\right]}{2}$
$=\frac{5^{\mathrm{fF}} / \mu \mathrm{m}^{2} \cdot 10 \mu \mathrm{~m} \cdot 0.1 \mu \mathrm{~m} \cdot[1 \mathrm{~V}+0.9 \mathrm{~V}]}{2}=4.75 \mathrm{fC}$
$\therefore$ Average channel charge per unit length $=\frac{\text { Average channel charge }}{L}=47.5^{\circ} \mathrm{fC} / \mu \mathrm{m}$
$I_{D}=($ Average channel charge per unit length $) \cdot($ velocity $)$
$\therefore I_{D}=47.5^{\mathrm{fC}} / \mu \mathrm{m} \cdot 10^{11} \mu \mathrm{~m} / \mathrm{s}=4.75 \mathrm{~mA}$

## Problem 3 [25 points]



The circuit shown above is biased so that all transistors are in saturation. Draw a small signal model (label all elements with appropriate symbols, e.g. $g_{m 1}, r_{o z}$ ) and find an algebraic expression for the small-signal voltage gain $a_{v}=v_{\text {out }} / v_{\text {in }}$ as a function of smallsignal parameters ( $g_{m}$ 's and $r_{o}$ 's). Use $g_{m} r_{o} \gg 1$ to simplify your result.

Small-signal model (neatness counts) [13 points]:


$$
a_{v}=-\mathbf{g}_{\mathbf{m} 1} / \mathbf{g}_{\mathrm{m} 2}
$$

$a_{v}=-g_{m 1} \cdot\left(r_{o 1} / / r_{o 2} / / \frac{1}{g_{m 2}}\right) \approx \frac{-g_{m 1}}{g_{m 2}}$

## Problem 4 [23 points]



The circuit shown above is biased so that the transistor is in saturation.
a) [8 points] What is the type of this amplifier?
Common Gate
b) [15 points] Find an algebraic expression for the small-signal voltage ratio $v_{2} / v_{1}$ for $i_{s}=0$ as a function of $\mathrm{R}_{1}, \mathrm{R}_{2}$, and transistor small-signal parameters.
Hint: you may find small-signal model very helpful to answer this question.

$$
v_{2} / v_{l}=\frac{\left(1+g_{m} r_{o}\right)}{r_{o}+R_{2}} \cdot R_{2}
$$

