UNIVERSITY OF CALIFORNIA, BERKELEY
MECHANICAL ENGINEERING
ME 106, FLUID MECHANICS
ODK/MIDTERM 2, FALL 2015

Last name: ____________________

First name: ____________________

Student ID: ____________________

Discussion: ____________________

Notes:

- You solution procedure should be legible and complete for full credit (use scratch paper as needed).

Question	Grade
1	
2	
3	
Total:	

1. ***(Total 15 points)*** Consider the unsteady, incompressible, 2D flow described by the velocity field

$$u = y^2 + t$$
$$v = x^2$$

(a) Find the equation, $x(y)$ or $y(x)$, for the streamline passing through point $(0, 0)$ at time $t = 0$.

(b) Find the equations, $x(t)$ and $y(t)$, describing a pathline originating at point $(0, 0)$ at time $t = 0$.

(c) Calculate the acceleration of a fluid particle at point $(0, 0)$ at time $t = 0$.

(a) Streamlines at $t = 0$ are given by relation

$$\frac{dx}{dy} = \frac{u}{v} = \frac{y^2}{x^2}$$

Integrating,

$$\int_{x_o}^{x} x^2 dx = \int_{y_o}^{y} y^2 dy$$

Carrying out the integral for $(x_0, y_0) = (0, 0)$ gives

$$\boxed{x = y}. \quad \textit{(5 point)}$$

(b) Pathline:

NOTE: The equations are nonlinearly coupled, making a closed form solution difficult, thus solutions should be left in integral form (if you noted this you received full credit). Alternatively, if you ignored this coupling in performing the integrals (as shown below), you also still received full credit.

For the x-coordinate

$$\frac{dx}{dt} = u = y^2 + t$$

Rewriting,

$$\int_{x_o}^{x} dx = \int_{t_o}^{t} (y^2 + t) dt$$

and carrying out the integral (ignoring y's dependence on t) for $x_0 = 0$ and $t_0 = 0$ we get

$$\boxed{x = y^2 t + \frac{t^2}{2}}. \quad \textit{(2.5 point)}$$

For the y-coordinate

$$\frac{dy}{dt} = v = x^2$$

Rewriting,

$$\int_{y_o}^{y} dy = \int_{t_o}^{t} x^2 dt$$

and carrying out the integral (ignoring x's dependence on t) for $y_0 = 0$ and $t_0 = 0$ we get

$$\boxed{y = x^2 t}. \quad \textit{(2.5 point)}$$

(c) Acceleration:

$$\frac{D\mathbf{v}}{Dt} = \frac{\partial \mathbf{v}}{\partial t} + u\frac{\partial \mathbf{v}}{\partial x} + v\frac{\partial \mathbf{v}}{\partial y}$$

$$a_x = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = 1 + x^2 2y$$

$$a_y = \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = (y^2 + t)(2x)$$

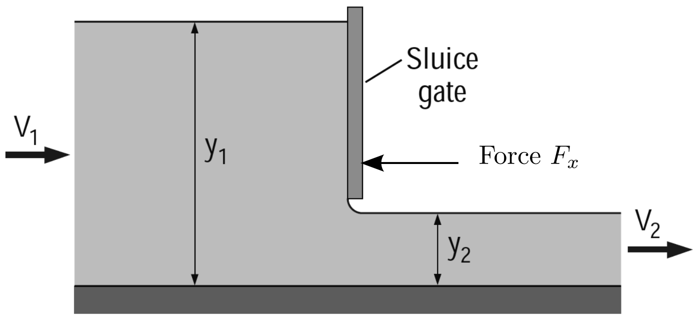
At $(x, y) = (0, 0)$ and $t = 0$, we get

$$\boxed{\mathbf{a} = (1, 0)}\,. \quad \textit{(5 point)}$$

2. ***(Total 20 points)*** A sluice gate of width b into the page controls the flow of water by raising or lowering a vertical plate. The water exerts a force $\mathbf{F}$ on the gate. Let ρ be the water density and other variables be as shown in the diagram. Disregarding the wall shear forces at the solid surfaces, and assuming steady, uniform flow:

   (a) Solve for the horizontal component of the force, F_x, the water imposes on the gate. Express answer in terms of $(\rho, y_1, y_2, b, g$ and $V_1)$

   (b) Based on the expression you derived above, derive an expression for y_2 when F_x is a maximum. Assume V_1 and y_1 remain constant.



(a) Define control volume as water only. Forces F_{p1} and F_{p2} due to pressures at sections (1) and (2) are given by

$$F_{p1} = P_{c1}A_1 = \rho g \frac{y_1}{2}(y_1 b) \quad \textit{(2 point)}$$

$$F_{p2} = P_{c2}A_2 = \rho g \frac{y_2}{2}(y_2 b) \quad \textit{(2 point)}$$

The momentum equation in the x direction gives

$$-F_x + F_{1p} - F_{p2} = -(\rho A_1 V_1)V_1 + (\rho A_2 V_2)V_2. \quad \textit{(5 point)}$$

Also from continuity $V_1 y_1 = V_2 y_2$. *(1 point)* Hence

$$\boxed{F_x = \rho b y_1 V_1^2 \left(1 - \frac{y_1}{y_2}\right) + \frac{1}{2}\rho g b \left(y_1^2 - y_2^2\right)}. \quad \textit{(5 point)}$$

This is the force of the gate on the water. The force on the gate is $-F_x$.

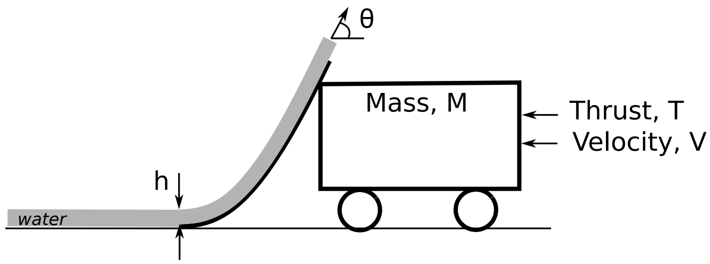
(b) F_x is maximum when

$$\frac{\partial F_x}{\partial y_2} = 0 = \frac{\rho b y_1^2 V_1^2}{y_2^2} - \rho g b y_2 \quad \textit{(5 point)}$$

3. ***(Total 20 points)*** A vehicle of mass M scoops stationary water of density ρ with depth h and width b into the page, creating an upward jet with angle θ. Assume the incoming and outgoing stream of water on the scoop have the same area. Neglect air drag, wheel friction and gravity effects.

   (a) Determine the thrust T to maintain a constant acceleration a in terms of the variables given.

   (b) Assume that the thrust is removed ($T = 0$), hence the vehicle decelerates from initial velocity V_0 at $t = 0$. Based on the expression you derived above, find the expression for the velocity $V(t)$ as a function of time (note: $a = \frac{dV}{dt}$).



(a) Choose control volume moving with the vehicle *(1 point)*. Since the absolute fluid velocity is zero, the relative velocity entering the control volume is $W = 0 - (-V)$ *(1 point)* hence the mass flux passing the control surface is $\dot{m} = \rho h b V$ *(2 point)*. Due to the conservation of mass, the exit velocity is also V *(1 point)*. The momentum equation in the x direction is

$$-T - Ma = -\dot{m}V + \dot{m}V\cos(\theta) \quad \textit{(6 point)}. \tag{1}$$

Hence

$$T = \rho h b V^2(1 - \cos(\theta)) - Ma. \quad \textit{(1 point)}$$

(b) Setting $T = 0$ in equation (1) and $a = \dot{V}$ yields

$$M\dot{V} = \rho b h V^2(1 - \cos(\theta)). \quad \textit{(2 point)}$$

Note, we have flipped the sign on Ma since vehicle is <u>decelerating</u> *(1 point)*. Let the constant $C = \rho b h(1 - \cos(\theta))/M$. Separating the equation

$$\int_{V_0}^{V} \frac{dV}{V^2} = \int_0^t C dt. \quad \textit{(2 point)}$$

Integration and simplifying for $V(t)$ yields

$$V(t) = \frac{V_0}{1 - V_0 C t}. \quad \textit{(3 point)}$$

Summary of Equations:
Chapter 4:

Equation for streamlines	$\frac{dy}{dx} = \frac{v}{u}$
Acceleration	$\mathbf{a} = \frac{\partial \mathbf{V}}{\partial t} + u\frac{\partial \mathbf{V}}{\partial x} + v\frac{\partial \mathbf{V}}{\partial y} + w\frac{\partial \mathbf{V}}{\partial z}$
Material derivative	$\frac{D(\ )}{Dt} = \frac{\partial(\ )}{\partial t} + (\mathbf{V}\cdot\nabla)(\ )$
Streamwise and normal components of acceleration	$a_s = V\frac{\partial V}{\partial s}, \qquad a_n = \frac{V^2}{\mathcal{R}}$
Reynolds transport theorem (restricted form)	$\frac{DB_{\text{sys}}}{Dt} = \frac{\partial B_{\text{cv}}}{\partial t} + \rho_2 A_2 V_2 b_2 - \rho_1 A_1 V_1 b_1$
Reynolds transport theorem (general form)	$\frac{DB_{\text{sys}}}{Dt} = \frac{\partial}{\partial t}\int_{\text{cv}} \rho b\, d\forall + \int_{\text{cs}} \rho b\, \mathbf{V}\cdot\hat{\mathbf{n}}\, dA$
Relative and absolute velocities	$\mathbf{V} = \mathbf{W} + \mathbf{V}_{\text{cv}}$

Chapter 5:

Conservation of mass	$\frac{\partial}{\partial t}\int_{\text{cv}} \rho\, d\forall + \int_{\text{cs}} \rho \mathbf{V}\cdot\hat{\mathbf{n}}\, dA = 0$
Mass flowrate	$\dot{m} = \rho Q = \rho A V$
Average velocity	$\overline{V} = \frac{\int_A \rho \mathbf{V}\cdot\hat{\mathbf{n}}\, dA}{\rho A}$
Steady flow mass conservation	$\sum \dot{m}_{\text{out}} - \sum \dot{m}_{\text{in}} = 0$
Moving control volume mass conservation	$\frac{\partial}{\partial t}\int_{\text{cv}} \rho\, d\forall + \int_{\text{cs}} \rho \mathbf{W}\cdot\hat{\mathbf{n}}\, dA = 0$
Deforming control volume mass conservation	$\frac{DM_{\text{sys}}}{Dt} = \frac{\partial}{\partial t}\int_{\text{cv}} \rho\, d\forall + \int_{\text{cs}} \rho \mathbf{W}\cdot\hat{\mathbf{n}}\, dA = 0$
Force related to change in linear momentum	$\frac{\partial}{\partial t}\int_{\text{cv}} \mathbf{V}\rho\, d\forall + \int_{\text{cs}} \mathbf{V}\rho\mathbf{V}\cdot\hat{\mathbf{n}}\, dA = \sum \mathbf{F}_{\substack{\text{contents of the}\\ \text{control volume}}}$
Moving control volume force related to change in linear momentum	$\int_{\text{cs}} \mathbf{W}\rho\mathbf{W}\cdot\hat{\mathbf{n}}\, dA = \sum \mathbf{F}_{\substack{\text{contents of the}\\ \text{control volume}}}$
Vector addition of absolute and relative velocities	$\mathbf{V} = \mathbf{W} + \mathbf{U}$