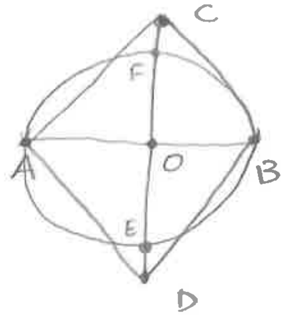
Instructor: Kathryn Mann

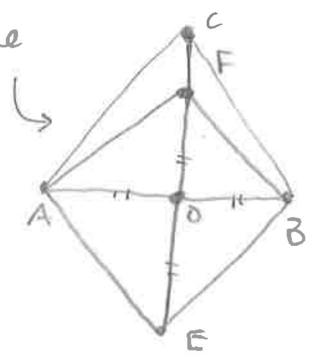
1a. Given A, B construct two equilateral triangles ABC, ABD, with base AB. Form line CD, this meets AB at O.



Draw a circle centered at O with radius OB.
This meets the line CD at E and F.

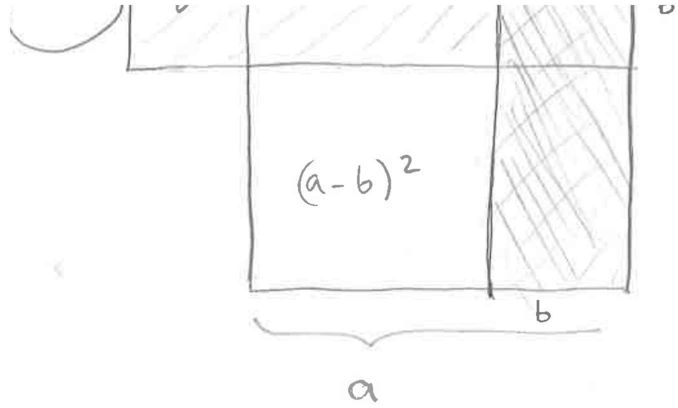
Form segments AF, FB, BE and EA. AFBE is a square.

1b. We have



from part 1a.

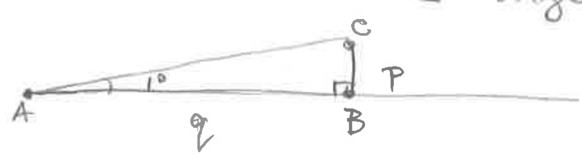
$|AO| = |OB| = |OE| = |OF|$ since all are radii of the circle.
Since $|CA| = |CB|$, by S-S-S applied to triangles COB and COA, we have $\measuredangle AOC = \measuredangle BOC$
so both are $90°$. Similarly, $\measuredangle AOE = \measuredangle BOE = 90°$
Now S-A-S applied to BOE, AOE, FOA, and FOB shows all these triangles are congruent, so all four sides of the square have the same length.
Also, $\measuredangle AEB = \measuredangle AEO + \measuredangle OEB = 2\measuredangle AEO = 2\cdot\left(\frac{180-90}{2}\right)$
$= 90°$. The same is true for since ΔAOE is



Total area is a^2+b^2.
Shaded regions each have area ab

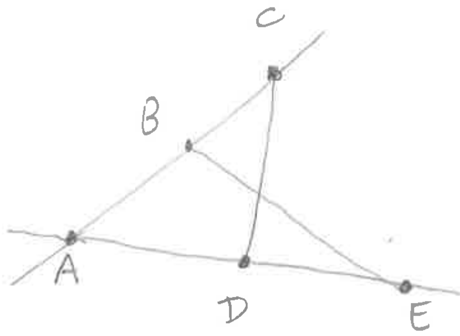
So $a^2+b^2 - 2ab = (a-b)^2$

3. Suppose for contradiction that $\tan(1°) = \frac{p}{q}$.
~~[illegible]~~ We can construct length q, a right angle and length p, therefore a $1°$ angle.



By repeating this constration using AC as the base, we can then construct $2°$, and inductively any angle that is an integer number of degrees.

But we proved in class that a $20°$ angle is not constructable.



A, B, E are not collinear, since A, B, C are but E lies on a diff't line than A and C.
Similarly A, D, C are not collinear.

Apply Pasch's axiom to triangle ABE, and line CD; conclude that CD intersects either segment $\overline{AB}$ or segment $\overline{BE}$. If it intersected $\overline{AB}$, then DC would have to lie on line AC (by I1) but A, D, and C are not collinear.

Thus, CD intersects $\overline{BE}$, say at X.

Apply Pasch's axiom now to ACD, and conclude as above that line BE intersects the segment i

say at Y.

Since X and Y both lie on BE and CD, they mus be the same point (by I1).
So $X = Y$ and this lies on $\overline{BE} \cap \overline{CD}$.