1. Consider the Op Amp circuit on the right.
   a. Derive the voltage gain expression of the amplifier if we consider the Op Amp to be ideal (infinite open-loop gain, infinite input resistance, zero output resistance)?
   b. What is the input resistance of the amplifier?
   c. What is the value of the voltage gain for a signal with 10 kΩ source resistance?

   a.)

   \[ V = 0 \Rightarrow V = 0 \]
   \[ i = \frac{V_o - 0}{1 \text{MΩ}} = \frac{V_i - 0}{10 \text{kΩ}} \]
   \[ \Rightarrow \frac{V_o}{V_i} = \frac{1 \text{MΩ}}{10 \text{kΩ}} = -100 \left( \frac{V}{V} \right) \]
   \[ \frac{V_o}{V_i} = -100 \left( \frac{V}{V} \right) \]

   b.)
   \[ i_i = \frac{V_i}{10 \text{kΩ}} \Rightarrow R_{in} = \frac{V_i}{i_i} = 10 \text{kΩ} \]
   \[ R_{in} = 10 \text{kΩ} \]

   c.)
   \[ V_o = \frac{V_e}{V_i} = A_V \left( \frac{R_{in}}{R_{in} + R_s} \right) = -100 \cdot 5 = -50 \]
   \[ \Rightarrow \frac{V_o}{V_i} = -50 \left( \frac{V}{V} \right) \]
2. For the same circuit as in Problem 1 and the same source resistance, but consider an Op Amp with a finite open loop gain of $A_0 = 100,000$, and an open-loop bandwidth of 100 Hz. The input resistance of the Op Amp is still infinite, and the output resistance is zero.

a. What is the 3-dB frequency of the amplifier in unit of Hz?

(You don't need to re-derive the expression if you can deduce the frequency response of the close-loop amplifier).

b. Show the frequency response in Bode plot. Mark the 3-dB frequency, unity gain frequency, the low frequency gain in dB, and the slope in dB/decade.
3. A non-inverting amplifier with variable gain is shown on the right. Here, $R_1 = 1 \, \text{k\Omega}$ and $R_2$ is variable from 100 k\Omega to 100 M\Omega. This input signal is:

$$v_i(t) = (10\, \text{mV}) \cdot \sin (2\pi \cdot 10\, \text{kHz} \cdot t).$$

The Op Amp has a slew rate (SR) of 6.28 V/\mu s.

a. First, assume the Op Amp has infinite bandwidth. If we gradually increase the gain of the amplifier by increasing $R_2$, at what value of $R_2$ does the output become slew rate limited?

b. Now consider the Op Amp with a finite gain-bandwidth product of 100 MHz and $R_2 = 20 \, \text{M\Omega}$, is the amplifier bandwidth-limited or SR-limited? Show the calculation supporting your answer.

\[ V_o = A_v \cdot v_i \]

\[ \frac{dV_o}{dt} = A_v \cdot 10\, \text{mV} \cdot 2\pi \cdot 10\, \text{kHz} \cdot \cos (2\pi \cdot 10\, \text{kHz} \cdot t) \]

\[ = A_v \cdot 10\, \text{mV} \cdot 2\pi \cdot 10\, \text{kHz} \leq \text{SR} = 6.28\, \text{V/\mu s} \]

\[ A_v \leq \frac{10^2}{0.1} = 10^4 \Rightarrow R_2 \leq 10\, \text{M\Omega} \]  

b. Given $R_2 = 20\, \text{M\Omega}$, $A_v = 1 + \frac{2\, \text{M\Omega}}{10\, \text{kHz}} = 2 \times 10^4$

\[ \frac{A_v}{f_{3dB}} = \frac{100\, \text{kHz}}{2 \times 10^4} = 5\, \text{kHz} < 10\, \text{kHz} \]

At first, the Op Amp is **BW limited**.

Then, consider the BW caused attenuation if answer SR limited (with math)

\[ A_v = \frac{2 \times 10^4}{\sqrt{1 + \left(\frac{f_{3dB}}{f_{10kHz}}\right)^2}} \]

\[ f_{10kHz} = 2 \times 10^4 \]

\[ \text{NO SR limited} \]
4. A pn junction in the reverse-bias configuration can be used as a variable capacitor. In the lab, we measured the capacitances of a pn junction diode at two voltages:
   (1) At 2.4 V reverse bias, the capacitance is measured to be 0.5 pF
   (2) At 12 V reverse bias, the capacitance is measured to be 0.25 pF
   a. Find the capacitance of the pn junction diode at zero bias.
   b. Find the built-in potential of the pn junction diode.

\[ G_{iv} : \quad C_{jo} = A \sqrt{\frac{\pi q}{2}} \frac{N_A N_D}{(N_A + N_D) V_0} = \frac{\alpha}{2 \sqrt{V_0}} \]

We know with reverse bias \( V_0 \gg V_R \)

\[ \Rightarrow C_j = \frac{\alpha}{2 \sqrt{V_0 + V_R}} \]

\[ \Rightarrow C_j = C_{jo} \cdot \sqrt{\frac{V_0}{V_0 + V_R}} = \frac{C_{jo}}{\sqrt{1 + \frac{V_R}{V_0}}} \]

\[ C_j^2 \left(1 + \frac{V_R}{V_0}\right) = C_{jo}^2 \]

\( V_R = 2.4 \text{V}, \quad C_j = 0.5 \text{ pF} \)

\( \Rightarrow 0.25 \left(1 + \frac{2.4}{V_0}\right) = C_{jo}^2 \)

\( V_R = 12 \text{V}, \quad C_j = 0.25 \text{ pF} \)

\( \Rightarrow 0.0625 \left(1 + \frac{12}{V_0}\right) = C_{jo}^2 = 0.25 \left(1 + \frac{12}{V_0}\right) \)

\[ 1 + \frac{12}{V_0} = 4 + \frac{7.6}{V_0} \Rightarrow V_0 + 12 = 4V_0 + 7.6 \]

\[ \Rightarrow 3V_0 = 2.4, \quad V_0 = 0.8 \text{V} \]
5. Consider the following two one-sided pn junction diodes with opposite doping profiles:
Diode A: \( N_D = 10^{18} \text{ cm}^{-3}, \ N_A = 10^{16} \text{ cm}^{-3} \)
Diode B: \( N_D = 10^{16} \text{ cm}^{-3}, \ N_A = 10^{18} \text{ cm}^{-3} \)
The rest of the material parameters are shown in the table. Other parameters (e.g., junction area) of the two diodes are identical.

(a) What is the ratio of the current going through the diodes under a forward bias voltage of 0.8V?
(b) What is the difference of the built-in voltages between these two diodes?
(c) What is the ratio of the diode capacitances at zero bias?

\[
\begin{align*}
\text{Diode A:} & \quad I_A = Aq \eta_i z \left( \frac{D_p}{L_p N_D} + \frac{D_n}{L_n N_A} \right) \left( e^{\frac{V}{kT}} - 1 \right) \\
\text{Diode B:} & \quad I_B = Aq \eta_i z \left( \frac{D_p}{L_p N_A} \right) \left( e^{\frac{V}{kT}} - 1 \right)
\end{align*}
\]

\[\Rightarrow \frac{I_A}{I_B} = \frac{D_n}{D_p} \cdot \frac{L_p}{L_n} \cdot \frac{N_D}{N_A} = \frac{30}{10} \cdot \frac{10^{16}}{10^{16}} = \frac{3}{2} \]

\[V_0 = V \ln \left( \frac{N_A N_D}{N_i^2} \right) \quad N_A N_D > 10^{16} \cdot 10^{16} = 10^{32} \quad \text{for both diodes}
\]

\[\Rightarrow V_{DA} = V_{DB} \Rightarrow V_{DA} - V_{DB} = 0 \text{V}
\]

\[C_{j0} = A \sqrt{\frac{\varepsilon}{2}} \frac{N_A N_D}{N_A^2 N_D} \cdot \frac{1}{1 + \frac{N_A N_D}{N_A + N_D}} = \frac{N_A N_D}{N_A + N_D} \]

\[\Rightarrow C_{j0A} = C_{j0B}
\]

\[\Rightarrow \frac{C_{j0A}}{C_{j0B}} = 1\]