## Second Midterm Examination

$\qquad$ Discussion Section \# $\qquad$
Signature $\qquad$ Discussion Section TA $\qquad$
Student ID\# $\qquad$

This exam is closed book, but you are allowed one $8.5^{\prime \prime} \times 11$ " (double-sided) page of handwritten notes. You may use a calculator, however NO wireless calculators are allowed. Anyone using a wireless calculator will forfeit their exam and automatically receive the score of zero. You may use $g=10 \mathrm{~m} / \mathrm{s}^{2}$.
a) Write your name, Discussion Section \#, GSI name and SID\# on the top of all materials you intend to hand in and want to be graded.
b) Remember to circle all of your final answers.
c) Express all numerical results to 3 significant figures. Cross out any work you decide is incorrect, with an explanation in the margin.

Read through the entire exam to start. Work to maximize your credit - try to obtain at least partial credit on every part of every problem. Do your work clearly so we can easily follow. Show all work, using the front and back sides of this exam paper. If you do not show relevant work for any part of a problem, you will not be awarded any credit, even if the answer is correct. If you recognize that an answer does not make physical sense and you do not have time to find your error, write that you know that the answer cannot be correct and explain how you know this to be true. (We will award some credit for recognizing there is an error.) Do not get bogged down in algebra - if you have enough equations to solve for your unknowns, box the equations, state how you would finish, and move on (you can go back and complete the algebra later if you have time). And if you have questions about the interpretation of a problem, please ask!

| Problem 1 |  |
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| 2 |  |
| 3 |  |
| 4 |  |
| 5 |  |
| 6 |  |
| TOTAL |  |

Total of 120 points (points for each part indicated in problem).

1. A solid sphere of mass $m$ and radius $r$ starts at rest at height $h$ and rolls without slipping down the straight track and into a circular loop of radius $R$.
a) 10 points. Find the minimum starting height $h$ so the sphere will not lose contact at the top of the loop. (The moment of inertia for a
 solid sphere is $2 / 5 \mathrm{mr}^{\wedge} 2$.)
b) 10 points. Identify all the forces acting on the sphere and find the work done by each force in reaching the bottom of the straight track.
2. A spaceship is placed into a circular orbit of radius $R$ and speed $v$ around a newly discovered star.
a) 5 points. Find the mass of the star.
b) 10 points. A second spaceship of unknown origin approaches from far away on an unpowered trajectory that passes close by in a trajectory nearly tangent to the first starship's orbit, at a speed of $0.5 v$ relative to the first starship. Is this starship coming from
 outside the star system or is it in an orbit gravitationally bound to the star? Explain your reasoning and provide quantitative proof.
3. A sudden impulse $J$ is applied to a billiard ball of mass $m$ and radius $r$ at a spot $0.4 r$ below the ball's center. Assume $J$ acts over a very short time.
a) 5 points. Find the translational and rotational velocity
 of the ball just after the impulse is applied.
b) 5 points. The ball immediately collides elastically with a second billiard ball (same mass and radius) that is at rest. Find the translational and rotational velocities for both balls just after the collision. Assume collision occurs in a very short time, and there is no friction between the balls.
c) 15 points. The coefficient of kinetic friction between the balls and the table is $\mu_{k}$. Solve for the translational and rotational motion of both balls as a function of time after the collision. (Hint: Consider when each ball stops accelerating.)
d) 5 points. Is total mechanical energy conserved? Explain your reasoning and provide quantitative proof.
4. Astronaut Sally's space tug has an initial total mass (including her) of 5500 kg of which $80 \%$ is fuel. It burns fuel at a rate of $100 \mathrm{~kg} / \mathrm{s}$ and exhausts its gas at a speed of $1 \mathrm{~km} / \mathrm{s}$.
a) 10 points. Assuming the tug starts at rest and there are no external forces, find the tug's
 initial acceleration, the acceleration at burnout, and the final velocity.
b) 10 points. The tug used all its fuel to match speed with the space station, but unfortunately ended up a distance $d=1$ meter from the docking port. Sally decides to push off from the front wall of the tug to launch herself backwards with velocity $v=10 \mathrm{~m} / \mathrm{s}$. Determine whether Sally's action will allow the tug to dock. The tug length is $L=11 \mathrm{~m}$ from wall to wall and its mass empty is $M=$ 1000 kg . Sally's mass (assume point mass) is $m=100 \mathrm{~kg}$. Explain your reasoning and provide quantitative proof.
5. 15 points. Ball 1 traveling with velocity $\overrightarrow{\mathbf{v}_{\mathbf{0}}}$ collides with ball 2 at rest and then with ball 3 at rest. All three balls have the same mass but their diameters can be different. The collisions can be elastic or inelastic, and there is no friction. For each part below, explain your reasoning and provide quantitative proof. After the collisions
a) Could ball 1 end up velocity $\overrightarrow{\mathbf{v}_{1}}=-\overrightarrow{\mathbf{v}_{\mathbf{0}}}$ ?

b) Could $\left|\theta_{2}\right| \neq\left|\theta_{3}\right|$ if ball 1 ends up at rest?
c) If ball 1 ends up going backwards ( $\overrightarrow{\mathbf{v}}_{1}$ antiparallel to $\overrightarrow{\mathbf{v}_{0}}$ ), could all three balls end up with the same speed $\left(\left|\overrightarrow{\mathbf{v}_{1}}\right|=\left|\overrightarrow{\mathbf{v}_{2}}\right|=\left|\overrightarrow{\mathbf{v}_{3}}\right|\right)$ but be going in different directions?
6. A block of mass $m_{1}$ at rest on a horizontal frictionless surface is connected to an unstretched spring with spring constant k whose other end is fixed. A second block of mass $m_{2}$ whose speed is $v$ collides with the first block.
a) 10 points. If the two blocks stick together after the one-dimensional collision, find the distance d by which the spring is compressed in terms of $m_{1}, m_{2}, v$, and $k$.
b) 5 points. Find the fraction of the initial kinetic energy that is lost as internal energy.
c) 5 points. If the collision occurs over a very short time $\Delta t$, find the average force on block $\mathrm{m}_{2}$.
