EE 120 SIGNALS AND SYSTEMS, Spring 2015
Midterm \# 1, February 25, Wednesday, 2:10-3:50 pm
Name $\qquad$

Important Instructions:
Closed book. Two letter-size cheatsheets are allowed.
Show all your work. An answer without explanation is not acceptable and does not guarantee any credit.
Only the front pages will be scanned and graded. If you need more space, please ask for extra paper instead of using the back pages.
Do not remove pages, as this disrupts the scanning. Instead, cross the parts that you don't want us to grade.

| Problem | Points | Score |
| :---: | :---: | :---: |
| 1 | 20 |  |
| 2 | 20 |  |
| 3 | 20 |  |
| 4 | 20 |  |
| 5 | 20 |  |
| Total | 100 |  |

1. (20 points) Consider a LTI system with impulse response:

$$
h(t)=(t+1) e^{-t} u(t) .
$$

a) (5 points) Determine if this system is causal.
b) (5 points) Determine if this system is stable.
c) (5 points) Determine the frequency response $H(j \omega)$.
d) (5 points) Write a differential equation relating the input $x(t)$ and output $y(t)$ of this system.

Additional workspace for Problem 1
2. a) ( 10 points) Show that the two LTI systems with impulse responses:

$$
h_{1}(t)=t e^{-t} u(t) \quad \text { and } \quad h_{2}(t)=-\frac{1}{2} \delta(t)+e^{-t} u(t)
$$

have the same output in response to the input $x(t)=\cos (t)$.
b) (10 points) Find another LTI system that gives the same response to $x(t)=\cos (t)$ as the two systems above and write its impulse response.

Additional workspace for Problem 2
3. (20 points) Given the period- 10 sequence $x[n]$ depicted below, determine the following quantities where $a_{k}$ denotes the $k$ th Fourier series coefficient:
a) $a_{0}$,
b) $a_{5}$
c) $\sum_{k=0}^{9} a_{k}$,
d) $\sum_{k=0}^{10} a_{k}$.


Additional workspace for Problem 3
4. (20 points) Consider a discrete-time LTI system with impulse response:

$$
h[n]= \begin{cases}5-|n| & -4 \leq n \leq 4 \\ 0 & \text { otherwise }\end{cases}
$$

a) (7 points) Show that $h[n]$ can be expressed as the convolution:

$$
h[n]=w[n] * w[n]
$$

where $w[n]$ is to be determined.
b) ( 7 points) Find the frequency response $H\left(e^{j \omega}\right)$.
c) (6 points) Determine if this system is generalized linear phase.

Additional workspace for Problem 4.
5. a) (5 points) Give a condition for the 2 D signal $x\left[n_{1}, n_{2}\right]$ that guarantees that the 2D Fourier transform $X\left(e^{j \omega_{1}}, e^{j \omega_{2}}\right)$ is real-valued.
b) (5 points) Give a condition for the 2 D signal $x\left[n_{1}, n_{2}\right]$ that guarantees that the 2D Fourier transform $X\left(e^{j \omega_{1}}, e^{j \omega_{2}}\right)$ satisfies:

$$
X\left(e^{j \omega_{1}}, e^{j \omega_{2}}\right)=X\left(e^{j \omega_{2}}, e^{j \omega_{1}}\right) \quad \text { for every } \omega_{1} \text { and } \omega_{2}
$$

c) (10 points) Calculate the frequency response of the 2 D moving average filter:

$$
y\left[n_{1}, n_{2}\right]=\frac{1}{9} \sum_{k_{1}=-1}^{1} \sum_{k_{2}=-1}^{1} x\left[n_{1}-k_{1}, n_{2}-k_{2}\right] .
$$

Additional workspace for Problem 5.

