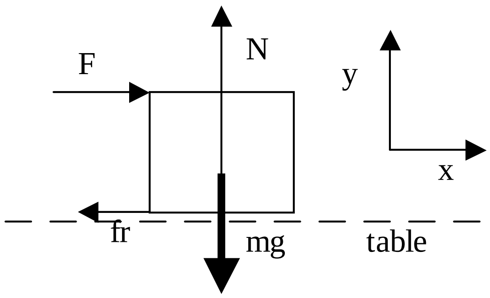
Final Prof Lanzara (Lecture 200)

Problem 1



a) The only external force acting on a block-table system is a horizontal force F.
Note: friction is an internal force for the block-table system.
External work comes from external force.

$$W_{external}=F_{\parallel}x=Fx$$

b) Energy dissipated by friction comes from work done by friction.

$$W_{fr} = \text{Energy lost}$$
$$fr = m\ N$$
$$F_{y,net} = N - mg = 0, \text{since the block in not accelerating in the y - direction.}$$
$$fr = m\ mg$$

The force of friction acts over distance x.

$$W_{fr} = -m\ mg\ x$$
$$\text{Energy lost} = -m\ mg\ x$$

c) Work-Kinetic energy theorem can be used to solve this part.

$$W_{box}=K.E.$$

$$W_{box} = (\text{Total force acting on the box}) \cdot x = (F - fr) \cdot x = (F - m\ mg)x$$

$$K.E. = \frac{1}{2} m\, v_{final}^2 - \frac{1}{2} m\, v_{initial}^2 = \frac{1}{2} m\, v_{final}^2, \text{since } v_{initial} = 0$$
$$\frac{1}{2} m\, v_{final}^2 = (F - m\ mg)x$$
$$v_{final} = \sqrt{\frac{2\,(F - m\ mg)\ x}{m}}$$

Problem 2. (Physics 7A FINAL-SECTION 2, Lanzara)

(a) The inertia moment for the hoop with radius $R = 0.08m$ and mass $m = 0.18kg$ becomes

$$I = mR^2 = 0.00152kgm^2$$

. Since there are two external forces acting on the hoop, T (tension, upward) and $F_g = mg$ (gravity, downward), the equation of motion for the vertical motion reads

$$ma = T - mg \tag{1}$$

and the equation of motion for the rotation becomes

$$I\alpha = RT \sin\left(-90°\right). \tag{2}$$

Because this is a rolling without slipping, the linear acceleration and angular acceleration are related by

$$a = R\alpha$$

, we can eliminate α from the equation (2) and solve for tension T by eliminating a,

$$T = mg + ma = mg - T \quad \rightarrow \quad T = \frac{mg}{2} = 0.882N. \tag{3}$$

(b) From the equations of motion found above, we have $a = -\frac{g}{2}$ and it is a constant acceleration motion. Thus, we can use the formula

$$y = y_0 + v_0 t + \frac{1}{2}at^2$$

and for $y = 0.75m$ down from the initial position, time t becomes

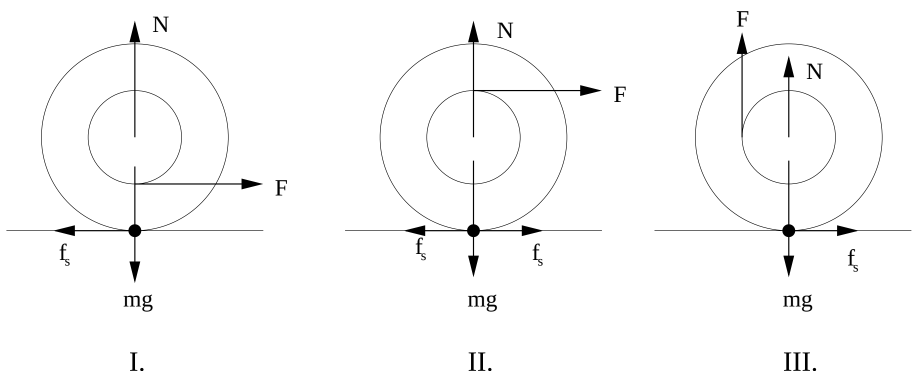
$$t = \sqrt{\frac{2\left(y_0 - y\right)}{-a}} = 0.553sec. \tag{4}$$

where $y_0 = 0$ and $v_0 = 0$.

(c) For the rolling without slipping, $v = R\omega$ and therefore

$$v = at = R\omega \quad \rightarrow \quad \omega = \frac{gt}{2R} = 33.87rad/sec.$$

(d) See figure 1. For the rolling without slipping, $a = R\alpha$:



There are two important points which we can choose as rotation axis, the center of the yo-yo and the contact point. The directions of rotation must be consistent for either choice of rotation and we will use this fact.

I. When we choose the contact point as the rotation axis, F is the only force that produces non-zero torque and it is clockwise.

If we choose the center of yo-yo as the rotation axis, F produces counterclockwise torque and therefore the friction f_s has to produce even larger torque clockwise. This tells that f_s must be directed to the left.

II. The direction of friction depends on the dimension of yo-yo:

if $mR > I$ then, f_s is to the right and vice versa.

III. If we take the contact point as the rotation axis, F is the only force that produces clockwise torque and this tells that $a > 0$. And there is only one horizontal force, friction f_s and this must be directed to the right.

(e) It rolls for all three cases because F produces clockwise torque which is the only torque when we choose the contact point as the rotation axis.

Problem #3

1) No external torque on the system.
therefore angular momentum is conserved.

$$L_o = L_f$$

$$\tfrac{1}{2} M R^2 \omega_o + m\left(\frac{R}{2}\right)^2 \omega_o = \tfrac{1}{2} M R^2 \omega_f + m R^2 \omega_f$$

$$\left(\frac{M}{2} + \frac{m}{4}\right)\omega_o = \left(\frac{M}{2} + m\right)\omega_f$$

$$\omega_f = \left(\frac{2M + m}{2M + 4m}\right)\omega_f = 4.66\,s^{-1}$$

2) No dissipative forces on the system.
therefore energy is conserved.

$$E_o = E_f$$

$$\tfrac{1}{2} I_d \omega_o^2 + \tfrac{1}{2} I_p \omega_o^2 = \tfrac{1}{2} I_d \omega_f^2 + \tfrac{1}{2} m v_{radial}^2 + \tfrac{1}{2} I_p \omega_f^2$$

$$\tfrac{1}{2} M R^2 \omega_o^2 + m R^2 \omega_o^2 = \tfrac{1}{2} M R^2 \omega_f^2 + m v_{radial}^2 + m R^2 \omega_f^2$$

$$\left(\frac{MR^2}{2} + mR^2\right)\omega_o^2 - \frac{MR^2}{2}\omega_f^2 = m v_{radial}^2 + m v_{tangential}^2$$

$$\left(\left(\frac{M}{2} + m\right)\omega_o^2 - M/2\,\omega_f^2\right) R^2/m = v_{radial}^2 + v_{tangential}^2 = v_{total}^2$$

$$v_{total} = 1.52\,m/s$$

Problem 4 - Lanzara Section 2

a) Simple harmonic Motion Amplitude: d $\omega = \sqrt{\frac{k}{m}}$

By the coordinate system shown:

$+x \leftarrow$

equilibrium $= 0$

$$x(t) = d\cos(\omega t)$$

$$v(t) = -\omega d \sin(\omega t)$$

where $\omega = \sqrt{\frac{k}{m}}$

b) $T = \frac{2\pi}{\omega} = \boxed{2\pi\sqrt{\frac{m}{k}}}$

c) $F = ma \Rightarrow -kx - bv = ma$

or $\boxed{m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = 0}$

$$\boxed{\omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}}$$

d) $P_0 = P_1 \Rightarrow mv_0 = 2mv_1 \Rightarrow |v_1| = \frac{|v_0|}{2}$ to the right

To find new amplitude: $E_0 = E_1$

$$\Rightarrow \tfrac{1}{2}kd^2 + \tfrac{1}{2}(2m)\underbrace{v^2}_{=\left(\frac{v_0}{2}\right)^2} = \tfrac{1}{2}kA^2$$

$$\Rightarrow \boxed{A = \sqrt{d^2 + \frac{1}{2}\frac{m}{k}v_0^2}}$$

& New Period: $\omega_{new} = \sqrt{\frac{k}{2m}}$ $\quad T = \frac{2\pi}{\omega}$

$$\Rightarrow \boxed{T_{new} = 2\pi\sqrt{\frac{2m}{k}}}$$

Prob 4 (con't)

e) Energy lost : $E_{lost} = k.E_f - k.E_i = \frac{1}{2}(2m)v_1^2 - \frac{1}{2}mv_0^2$

$= \frac{1}{2}2m\left(\frac{v_0}{2}\right)^2 - \frac{1}{2}mv_0^2 = -\frac{1}{4}mv_0^2$

$\Rightarrow \boxed{|E_{lost}| = \frac{1}{4}mv_0^2}$

f) Now at the time of collision m_A is at equilibrium point which means it has its maximum speed.
By part (a) this speed is $d\omega = d\sqrt{\frac{k}{m}}$ to the right. let call this v_A:

$P_0 = P_1 \Rightarrow m\left(v_0 + d\sqrt{\frac{k}{m}}\right) = 2\cancel{m}v_1$

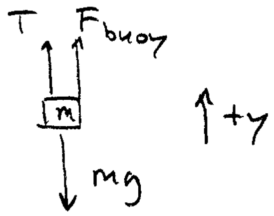
$\Rightarrow \boxed{v_1 = \frac{1}{2}\left(v_0 + d\sqrt{\frac{k}{m}}\right)}$ to the right

Energy lost: $E_{lost} = k.E_f - k.E_i$

$= \frac{1}{2}(2m)v_1^2 - \frac{1}{2}mv_0^2$

$\boxed{= \frac{1}{2}m\left[\frac{1}{2}\left(v_0 + d\sqrt{\frac{k}{m}}\right)^2 - v_0^2\right] - \frac{1}{2}kd^2}$

Problem 5



(a) $\Sigma F_y = F_{buoy} + T - mg = 0 \qquad F_{buoy} = \rho V g$

$\rho V g + T - mg = 0$

$$\boxed{V = \frac{mg - T}{\rho g}}$$

(b) $\Sigma F_y = F_{buoy} + T_b - mg = m(g/3)$ $\quad$ * Note: T_b not the same T in (a)

$T_b = \frac{4}{3} mg - F_{buoy} = \frac{4}{3} mg - \rho g V$ $\quad$ use V from (a)

$$\boxed{T_b = T + \tfrac{1}{3} mg}$$

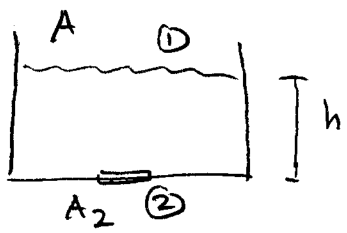
(c) $F_{buoy} + T_c - mg = m(-g/3)$

$$\boxed{T_c = T - \tfrac{1}{3} mg}$$

(d) $F_{buoy} + T_d - mg = -mg$

$T_d = -F_{buoy} \Rightarrow$ since tension can't be negative in the rope, we know $\boxed{T_d = 0}$

(e)



(i) $\rho g h + \frac{1}{2}\rho v_1^2 = \frac{1}{2}\rho v_2^2$ $\qquad$ (ii) $A v_1 = A_2 v_2$

$2gh = v_2^2 - v_1^2$ $\qquad$ $v_2 = \frac{A}{A_2} v_1$

(iii) $v_1 = \frac{-dh}{dt}$ $\quad$ note sign

plug (ii) into (i) $\Rightarrow 2gh = v_1^2\left(\frac{A^2}{A_2^2} - 1\right)$

simplify and plug in (iii) $\Rightarrow -\frac{dh}{dt} = \sqrt{\frac{2g A_2^2 h}{A^2 - A_2^2}}$

let $k = \sqrt{\frac{2g A_2^2}{A^2 - A_2^2}} \Rightarrow -\frac{dh}{\sqrt{h}} = k\,dt$

integrate w/ initial height h_0 and final time t_f

$$-\int_{h_0}^{0} \frac{dh}{\sqrt{h}} = \int_0^{t_f} k\,dt$$

$$-2\sqrt{h}\,\Big|_0^{h_0} = kt\,\Big|_0^{t_f}$$

$$\boxed{t_f = \sqrt{\frac{2h}{g}\left(\frac{A^2 - A_2^2}{A_2^2}\right)}}$$